First Name: _____ Last Name: _____

Student-No: _____ Section: _____

Grade:

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Indefinite Integrals

1. [9 marks] Each part is worth 3 marks. Please write your answers in the boxes.

   (a) Calculate the indefinite integral $\int e^{-x}\sqrt{1+e^{-x}}\,dx$.

   Answer:

   (b) Calculate the indefinite integral $\int (x+1)e^{-x}\,dx$ for $x > 0$.

   Answer:

(c) (A Little Harder): Calculate the indefinite integral $\int \tan^5(x)\sec^3(x)\,dx$.

Answer:

Definite Integrals

2. [12 marks] Each part is worth 4 marks. Please write your answers in the boxes.

(a) Calculate $I = \int_0^{\pi/8} \sin^2(2x)\, dx$.

Answer:

(b) Calculate $I = \int_1^e x^2 \ln x\, dx$.

Answer:

(c) (A Little Harder): Calculate $I = \int_0^\infty e^{-x} \sin(x)\, dx$.

Answer:

Riemann Sum, FTC, and Volumes

3. [12 marks] Each part is worth 4 marks. Please write your answers in the boxes.

   (a) Calculate the infinite sum

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{8i}{n^2} e^{-4i^2/n^2}$$

   by first writing it as a definite integral. Then, **evaluate this integral**.

Answer:

   (b) For $x > 0$ define $F(x) = \int_1^x t^{-1/2}\, dt$ and $g(x) = \sqrt{F(x^2)}$. Calculate $g'(2)$.

Answer:

(c) Write a definite integral, with specified limits of integration, for the volume obtained by revolving the bounded region between $x = (y-2)^2$ and $x = 2-(y-2)^2$ about the vertical line $x = -2$. **Do not evaluate the integral**.

Answer:

4. (a) [2 marks] Plot the finite area enclosed by $y^2 = 4 - x$ and $x = 3y - 6$.

(b) [4 marks] Write a definite integral with specific limits of integration that determines this area. **Do not evaluate the integral**.

Answer:

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5. A solid has as its base the region in the xy-plane between $y = 1 - x^2/36$ and the x-axis. The cross-sections of the solid perpendicular to the x-axis are squares.

   (a) [4 marks] Write a definite integral that determines the volume of the solid.

   (b) [2 marks] **Evaluate the integral** to find the volume of the solid.

Answer:

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