First Name: _____ Last Name: _____

Student-No: _____ Section: _____

Grade:

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Indefinite Integrals

1. 9 marks Each part is worth 3 marks. Please write your answers in the boxes.

   (a) Calculate the indefinite integral $\int \sin^3(x)\,dx$.

   Answer:

   (b) Calculate the indefinite integral $\int \frac{1}{x(\ln x)^2}\,dx$ for $x > 0$.

   Answer:

(c) (A Little Harder): Calculate the indefinite integral $\int \frac{\sqrt{x^2-25}}{x} \, dx$ for $x > 5$.

Answer:

VERSION C

Definite Integrals

2. [12 marks] Each part is worth 4 marks. Please write your answers in the boxes.

(a) Calculate $\int_0^{\pi/8} \tan^5(2x) \sec^2(2x)\, dx$.

Answer:

(b) Calculate $\int_{-2}^{-1} \frac{1}{(x+2)^2+1}\, dx$.

Answer:

VERSION C

(c) (A Little Harder): Calculate $\int_0^1 x^3 \sqrt{1 - x^2}\, dx$.

Answer:

Riemann Sum, FTC, and Volumes

3. ⟨12 marks⟩ Each part is worth 4 marks. Please write your answers in the boxes.

(a) Calculate the infinite sum

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2i}{n^2} e^{-i^2/n^2}$$

by first writing it as a definite integral. Then, **evaluate this integral.**

Answer:

(b) Define $F(x)$ and $g(x)$ by $F(x) = \int_0^x e^{-t}\, dt$ and $g(x) = \sqrt{F(x^2)}$. Calculate $g'(2)$.

Answer:

(c) Write a definite integral, with specified limits of integration, for the volume obtained by revolving the bounded region between $y = (x-2)^2$ and $y = 2 - (x-2)^2$ about the horizontal line $y = -2$. **Do not evaluate the integral**.

Answer:

$$V = \pi \int_{1}^{3} \left[\left(4 - (x-2)^2\right)^2 - \left((x-2)^2 + 2\right)^2 \right] \, dx$$

4. (a) $\boxed{\text{2 marks}}$ Plot the finite area enclosed by $4y^2 = 8 - x$ and $y = x/4$.

(b) $\boxed{\text{4 marks}}$ Write a definite integral with specific limits of integration that determines this area. **Do not evaluate the integral**.

Answer:

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5. A solid has as its base the region in the xy-plane between $y = 1 - x^2/16$ and the x-axis. The cross-sections of the solid perpendicular to the x-axis are isosceles right triangles (i.e. $45 - 45 - 90$ triangles) with the longest side (i.e. the hypoteneuse) in the base.

    (a) 4 marks Write a definite integral that determines the volume of the solid.

    Answer:

    (b) 2 marks **Evaluate the integral** to find the volume of the solid.

    Answer:

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