First Name: _____ Last Name: _____

Student-No: _____ Section: _____

Grade:

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Indefinite Integrals

1. [9 marks] Each part is worth 3 marks. Please write your answers in the boxes.

   (a) Calculate the indefinite integral $\int \frac{3x}{x+4}\,dx$.

   Answer:

   (b) Calculate the indefinite integral $\int \arctan(x)\,dx$.

   Answer:

(c) (A Little Harder): Calculate the indefinite integral $\int \frac{1}{x\sqrt{x^2-1}}\,dx$ for $x > 1$.

Answer:

Definite Integrals

2. 12 marks Each part is worth 4 marks. Please write your answers in the boxes.

(a) Calculate $\int_0^{\pi/4} \tan^2(x)\,dx$

Answer:

(b) Calculate $\int_{-\pi}^{\pi} (1+x^3)\cos^2(x)\,dx$.

Answer:

(c) (A Little Harder): Calculate $\int_0^\infty e^{-x}\cos(x)\,dx$.

Answer:

Riemann Sum, FTC, and Volumes

3. [12 marks] Each part is worth 4 marks. Please write your answers in the boxes.

   (a) Calculate the infinite sum

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{8i}{n^2} \sqrt{1 + \frac{4i^2}{n^2}}$$

   by first writing it as a definite integral. Then, **evaluate this integral**.

Answer:

   (b) Define $F(x)$ and $g(x)$ by $F(x) = \int_0^x \cos^2(t)\, dt$ and $g(x) = x\, F(x^2)$. Calculate $g'(\sqrt{\pi})$.

Answer:

(c) Write a definite integral, with specified limits of integration, for the volume obtained by revolving the bounded region between $y = x^2$ and $y = 9x$ about the horizontal line $y = -2$. **Do not evaluate the integral**.

Answer:

4. (a) [2 marks] Plot the finite area enclosed by $y^2 = 10 - x$ and $x = (y-2)^2$.

(b) [4 marks] Write a definite integral with specific limits of integration that determines this area. **Do not evaluate the integral**.

Answer:

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5. A solid has as its base the region in the xy-plane between $y = 1 - x^2/16$ and the x-axis. The cross-sections of the solid perpendicular to the x-axis are semi-circles with the diameter of the semi-circle in the base.

   (a) [4 marks] Write a definite integral that determines the volume of the solid.

   Answer:

   (b) [2 marks] **Evaluate the integral** to find the volume of the solid.

   Answer:

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