First Name: _____ Last Name: _____

Student-No: _____ Section: _____

Grade:

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VERSION A

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Indefinite Integrals

1. 9 marks Each part is worth 3 marks. Please write your answers in the boxes.

(a) Calculate the indefinite integral $\int x^2\sqrt{8 - x^3}\,dx$ for $x < 2$.

Answer:

(b) Calculate the indefinite integral $\int x\sqrt{x - 1}\,dx$ for $x > 1$.

Answer:

(c) (A Little Harder): Calculate the indefinite integral $\int \ln\left(1+x^2\right)\,dx$.

Answer:

Definite Integrals

2. 12 marks Each part is worth 4 marks. Please write your answers in the boxes.

(a) Calculate $\int_0^\pi \sin^3(x)\,dx$.

Answer:

(b) Calculate $\int_{-1}^1 \left(x^2 e^{-x^3} + x^5 \cos(x) \right) dx$.

Answer:

(c) (A Little Harder): Calculate $\int_1^e (\ln x)^2 \, dx$.

Answer:

Riemann Sum, FTC, and Volumes

3. ☐ 12 marks ☐ Each part is worth 4 marks. Please write your answers in the boxes.

(a) Calculate the infinite sum

$$\lim_{n \to \infty} \sum_{i=1}^{n} \frac{2i}{n^2 \left(4 + i^2/n^2\right)}$$

by first writing it as a definite integral. Then, **evaluate this integral**.

> Answer:

(b) Define $F(x)$ and $g(x)$ by $F(x) = \int_1^x \ln t \, dt$ and $g(x) = x \, F(x^2)$ for $x > 1$. Calculate $g'(e)$.

> Answer:

(c) Write a definite integral, with specified limits of integration, for the volume obtained by revolving the bounded region between $y = x^2$ and $y = 6x - 5$ about the horizontal line $y = -2$. **Do not evaluate the integral**.

Answer:

4. (a) ⏐2 marks⏐ Plot the finite area enclosed by $y^2 = 6 + x$ and $2y = x - 2$.

(b) ⏐4 marks⏐ Write a definite integral with specific limits of integration that determines this area. **Do not evaluate the integral**.

Answer:

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5. A solid has as its base the region in the xy-plane between $y = 1 - x^2/16$ and the x-axis. The cross-sections of the solid perpendicular to the x-axis are isosceles right triangles (i.e. $45 - 45 - 90$ triangles) with the longest side (i.e. the hypoteneuse) in the base.

(a) 4 marks Write a definite integral that determines the volume of the solid.

Answer:

(b) 2 marks **Evaluate the integral** to find the volume of the solid.

Answer:

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