MOD p REPRESENTATIONS OF p-ADIC GL_2 AND CŒFFICIENT SYSTEMS ON THE TREE

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Abstract. — Let F be a p-adic field with uniformizer π. We consider the universal module of the functions with value in $\overline{\mathbb{F}}_p$ and finite support on the cosets $I(1)\pi^{\mathbb{Z}}\backslash \mathrm{GL}_2(F)$, where $I(1)$ denotes the pro-p-Iwahori subgroup of $\mathrm{GL}_2(F)$. We show that, as a smooth representation of $\mathrm{GL}_2(F)$, it is isomorphic to the homology of level 0 of the associated cœfficient system on the tree. We then use the result of [**O2**] to show that any representation of $\mathrm{GL}_2(\mathbb{Q}_p)$, with a central character and generated by its pro-p-invariant subspace, is the homology of level 0 of a cœfficient system on the tree.

The construction by Colmez of an explicit (ϕ, Γ)-module associated to an irreducible mod p representation of $\mathrm{GL}_2(\mathbb{Q}_p)/p^{\mathbb{Z}}$ lies on the existence of a *standard presentation* for such a representation [**C**]. In other words, the representation is the homology of level 0 of a cœfficient system on the tree whose fibers are finite dimensional vector spaces. This result has been proved by Colmez by direct calcultation of a *standard presentation* for an irreducible mod p representation of $\mathrm{GL}_2(\mathbb{Q}_p)/p^{\mathbb{Z}}$. Vignéras has shown the existence of such a presentation by first considering the characteristic zero case and by reducing integral structures [**V**]. We show here the result generically and directly in characteristic p by first considering the case of the pro-p-universal module of GL_2 over any p-adic field. Theorem 2.1 of this paper says that this universal module is isomorphic to the homology of level 0 of the cœfficient system on the tree naturally associated by Schneider-Stuhler ([**S-S**]). We then use [**O2**] to deduce results about the representations of $\mathrm{GL}_2(\mathbb{Q}_p)$.

1. The pro-p-universal module of $\mathrm{GL}_2(F)$.

We denote by F a p-adic field with ring of integers $\mathscr{O}$, maximal ideal $\mathscr{P}$, and residue field $\mathbb{F}_q$. We choose and fix a uniformizer π. We consider the p-adic group $G = \mathrm{GL}_2(F)/\pi^{\mathbb{Z}}$. We see the maximal compact $K = \mathrm{GL}_n(\mathcal{O})$ and its congruence subgroup K_i, $i \in \mathbb{N}$, of the matrices congruent to $1 \mod \mathcal{P}^i$ as subgroups of G. Let I denote the standard upper Iwahori subgroup of K and

$I(1)$ its unique pro-p-Sylow. Let $s = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and $\varpi = \begin{pmatrix} 0 & 1 \\ \pi & 0 \end{pmatrix}$. The latter normalizes the Iwahori subgroup and ϖ^2 is central.

Let $\mathbf{G}$ denote the finite group $\mathrm{GL}_2(\mathbb{F}_q)$ and $\mathbf{B}$ the upper triangular Borel subgroup with Levi decomposition $\mathbf{UT}$. The double cosets $\mathbf{U}\backslash\mathbf{G}/\mathbf{U}$ are represented by the finite (extended) Weyl group $W_0^{(1)}$ of $\mathbf{G}$ which is the semi-direct product $\mathcal{S}_2.\mathbf{T}$, where $\mathcal{S}_2$ is viewed as a subgroup of $\mathbf{G}$. We fix an algebraic closure $\overline{\mathbb{F}}_p$ of $\mathbb{F}_q$. Let $\mathbf{C}$ denote what we call the *finite universal module* $\overline{\mathbb{F}}_p[\mathbf{U}\backslash\mathbf{G}]$. The $\overline{\mathbb{F}}_p$-algebra $\mathbf{H}$ of the $\overline{\mathbb{F}}_p[\mathbf{G}]$-endomorphisms of $\mathbf{C}$ will be called the *finite Hecke algebra*. By Frobenius, a basis of $\mathbf{H}$ identifies with the characteristic functions of the double cosets $\mathbf{U}\backslash\mathbf{G}/\mathbf{U}$.

Let C denote the pro-p-universal module $\overline{\mathbb{F}}_p[I(1)\backslash G]$. The $\overline{\mathbb{F}}_p$-algebra of the $\overline{\mathbb{F}}_p[G]$-endomorphisms of C is called the *pro-p-Hecke algebra* and denoted by $\mathcal{H}$. The subspace of C of the functions with support in K identifies with the finite universal module $\mathbf{C} = \overline{\mathbb{F}}_p[\mathbf{U}\backslash\mathbf{G}]$. Among the $\overline{\mathbb{F}}_p[G]$-endomorphisms of C, those stabilizing the subspace $\mathbf{C}$ constitute a subalgebra which identifies with the finite Hecke algebra. A basis of $\mathcal{H}$ is given by the characteristic functions of the double cosets $I(1)\backslash G/I(1)$ which are indexed by the semi-direct product $W^{(1)} =< \varpi > .W_0^{(1)}$. (Recall that $\mathbf{T}$ identifies with a sugroup of G *via* the Teichmüller isomorphism). It is an extended affine Weyl group with length function ℓ extending the length of the finite Coxeter group $\mathcal{S}_2$. The elements ϖ and $t \in T$ have length 0 ([**L**]). We denote by T_w the element of $\mathcal{H}$ corresponding to the double coset of $w \in G$. The following set is a basis of $\mathcal{H}$

$$\big\{(T_{s\varpi})^k T_t,\ \ T_\varpi(T_{s\varpi})^k T_t,\ \ (T_{s\varpi})^k T_s T_t,\ \ T_\varpi(T_{s\varpi})^k T_s T_t,\ \ k \in \mathbb{N},\ t \in \mathbf{T}\big\}.$$

Proposition 1.1. — *There is a system $\mathcal{D}$ of representatives of the right cosets $W^{(1)}/W_0^{(1)}$ such that*

$$\ell(dw_0) = \ell(d) + \ell(w_0), \quad \forall w_0 \in W_0^{(1)}.$$

Proof. — True and to be written for $\mathrm{GL}_n(F)$. Pour $n = 2$, one can choose $\mathcal{D} = \{(s\varpi)^k,\ \varpi(s\varpi)^k\}$.

□

Corollary 1.2. — *The pro-p-Hecke algebra $\mathcal{H}$ is a free module over the finite Hecke algebra $\mathbf{H}$ with basis $\{T_d\}_{d\in D}$.*

A basis of the universal module C is given by the characteristic functions of the cosets $I(1)\backslash G$ which we denote by

$$\big\{[I(1)w] = w^{-1}[I(1)],\ w \in I(1)\backslash G\big\}.$$

More generally, for $I(1)X$ a $I(1)$-homogeneous subset of G we denote by $[I(1)X]$ the corresponding characteristic function in C.

Proposition 1.3. — *For $i \in \mathbb{N}$, the K_{i+1}-invariant subspace $C^{K_{i+1}}$ is generated as an $\mathcal{H}$-module by the functions $\{[I(1)w],\ w \in I(1)\backslash G\}$ which are K_{i+1}-invariant.*

Proof. — By induction on i, see [**O2**, Proposition 2.3] (recent version). □

Corollary 1.4. — *The K_1-invariant subspace C^{K_1} of C is generated as an $\mathcal{H}$-submodule by $\mathbf{C}$. More precisely, as a vector space, it decomposes into the following direct sum :*

$$C^{K_1} = \bigoplus_{d \in D} T_d\, \mathbf{C}.$$

Proof. — [**O2**, Corollaire 2.4].

□

Remark 1.5. — The space $T_d\mathbf{C}$ is isomorphic to $\mathbf{C}$ because the restriction of T_d to the space of $I(1)$-invariants $\mathbf{C}^{I(1)} = \mathrm{Vect}\{[I(1)], T_s[I(1)]\}$ is injective.

Let $\mathcal{T}$ be the Bruhat-Tits tree of G. Let σ_0 denote the vertex corresponding to the lattice $\mathcal{O} \oplus \mathcal{O}$ in the canonical basis, whose stabilizer is K. The vertex $\varpi\sigma_0$ corresponds to the lattice $\mathcal{O} \oplus \varpi\mathcal{O}$.

Let $[\,.\,] : \mathbb{F}_q \to \mathcal{O}$ denote the Teichmüller lifting map. We see the elements of $\mathcal{O}$ as vectors in $\mathbb{F}_q^{\mathbb{N}}$ *via* the isomorphism $\tau : (x_j)_{j\in\mathbb{N}} \longmapsto \sum_{j\in\mathbb{N}} \pi^j [x_j]$.
Let $i \in \mathbb{N}$, $i \geq 1$, and $a \in \mathbb{F}_q^i$. Define

$$g_a^0 = \begin{pmatrix} -\tau(a) & \pi^{i-1} \\ 1 & 0 \end{pmatrix}, \qquad g_a^1 = \begin{pmatrix} 1 & 0 \\ -\pi\tau(a) & \pi^i \end{pmatrix}.$$

For convenience of notations in the proof of proposition 2.1, we set $g_\infty^0 = 1$. The set of outpointing edges of the tree is the set of the images of the oriented edge $(\sigma_0, \varpi\sigma_0)$ by the elements

$$\{1\,, g_a^\epsilon,\ \text{ for } \epsilon \in \{0,1\},\ a \in \mathbb{F}_q^i,\ i \geq 1\}.$$

For $g \in G$, we say that g is at distance $i \in \mathbb{N}$ if the vertex $g\sigma_0$ is at distance i from σ_0. It only depends on the class of g in G/K. A set of representatives of G/K is given by the "ends of the outpointing edges"

$$\{1, \varpi, g_a^\epsilon\varpi,\ \text{ for } \ \epsilon \in \{0,1\}, a \in \mathbb{F}_q^i, i \geq 1\}. \tag{1}$$

For $i \geq 1$, the i-radius sphere is $S_i = \{(g_a^0\varpi)\,\sigma_0,\ \ (g_b^1\varpi)\,\sigma_0,\ \text{for } a \in \mathbb{F}_q^i,\ b \in \mathbb{F}_q^{i-1}\}$.

Remark 1.6. — This system of representatives has the following "good" property : translation of the 1-radius sphere by the vertices at distance i is equal to the $i+1$-radius sphere.

Proposition 1.7. — *For $i \geq 1$, we have an isomorphim of modules over the pro-p-Hecke algebra :*

$$C^{K_{i+1}}/C^{K_i} \simeq \bigoplus_{g\in(1), g \text{ at distance } i} g(C^{K_1}/C^{I(1)}) \tag{2}$$

Proof. — [**O2**, Proposition 2.7](recent version). □

2. Mod p-representations of G and 0-homology of cœfficient systems on the tree.

The aim of this section is to show that the augmented complex of oriented chains of the cœfficient system associated to the universal module $\overline{\mathbb{F}}_p[I(1)\backslash G]$ is exact. If $F = \mathbb{Q}_p$ we will deduce the existence of a *standard presentation* for the representations of G generated by their $I(1)$-invariant subspace.

We denote by $\mathcal{T}(0)$ (resp. $\mathcal{T}(1)$) the set of vertices (resp. oriented edges) of the associated Bruhat-Tits tree.

2.1. Review of the G-equivariant homological cœfficient system associated to a smooth representation of G.— Following [**S-S**, II.2], we consider the (homological) coefficient system $\mathcal{V}$ associated to a smooth $\overline{\mathbb{F}}_p$-representation V of G. For each simplex σ, the space V_σ is equal to the space V^{U_σ} of invariants under the pro-unipotent radical of the parahoric subgroup associated to σ. The transition maps are the inclusions. Recall that we have called σ_0 the vertex of the tree corresponding to the lattice $\mathcal{O} \oplus \mathcal{O}$ in the canonical basis. The stabilizer $\mathfrak{K}(\sigma_0)$ of σ_0 is the maximal compact K. Denote by σ_1 the non-oriented edge $\{\sigma_0, \varpi\sigma_0\}$. Its stabilizer $\mathfrak{K}(\sigma_1)$ is the subgroup generated by ϖ and I, so that

$$V_{\sigma_0} = V^{K_1},\ \ V_{\sigma_1} = V^{I(1)}.$$

The space of 0-chains $Ch_0(\mathcal{V})$ is the set of functions

$$\mathcal{T}(0) \to \bigoplus_{\sigma\in\mathcal{T}(0)} V_\sigma$$

with finite support, sending a vertex σ to an element of V_σ with the natural action of G : under the action of $g \in G$, the function with support σ and value $v \in V_\sigma$ becomes the function with support $g\sigma$ and value $gv \in gV_\sigma = V^{gU_\sigma g^{-1}} = V^{U_{g\sigma g^{-1}}} = V_{g\sigma}$. The G-space of 0-chains is isomorphic to the compactly induced representation $ind_{\mathfrak{K}(\sigma_0)}^G V_{\sigma_0}$. We will denote by $[g, v]$ the element of $ind_{\mathfrak{K}(\sigma_0)}^G V_{\sigma_0}$

with support $\mathfrak{K}(\sigma_0)g^{-1}$ and value $v \in V_{\sigma_0}$ at g. It corresponds to the 0-chain $g\sigma_0 \mapsto gv$. Note that $[g,v] = g.[1,v]$. From now on we identifiy the 0-chains with this induced representation.

The space of oriented 1-chains $Ch_1(\mathcal{V})$ is the set of functions

$$\psi : \mathcal{T}(1) \to \bigoplus_{\{\sigma,\sigma'\}\text{non-oriented edge}} V_{\{\sigma,\sigma'\}}$$

with finite support, sending an oriented edge (σ,σ') to an element of the space $V_{\{\sigma,\sigma'\}}$ and satisfying $\psi(\sigma,\sigma') = -\psi(\sigma',\sigma)$. This space is endowed with the natural action of G and is isomorphic to the compactly induced representation $ind_{\mathfrak{K}(\sigma_1)}^G V_{\sigma_1} \otimes \epsilon$ where $\epsilon : \mathfrak{K}(\sigma_1) \to \overline{\mathbb{F}}_p^{\,*}$ is trivial on I and $\epsilon(\varpi) = -1$. The element of $ind_{\mathfrak{K}(\sigma_1)}^G V_{\sigma_1} \otimes \epsilon$ with support $\mathfrak{K}(\sigma_1)g^{-1}$ and value $v \in V_{\sigma_1}$ is denoted by $[[g,v]]$. It corresponds to the 1-chain with support $g\sigma_1$ sending the oriented edge $g(\sigma_0, \varpi\sigma_0)$ to gv. We identify the space of oriented 1-chains with this induced representation.

The G-equivariant boundary map, defined by

$$\begin{array}{rcl} \partial : Ch_1(\mathcal{V}) & \longrightarrow & Ch_0(\mathcal{V}) \\ [[1,\varpi v]] & \longmapsto & [1,\varpi v] - [\varpi, v], \end{array}$$

for all $v \in V^{I(1)}$, gives rise to a G-equivariant exact complex

$$0 \longrightarrow H_1(\mathcal{V}) \longrightarrow Ch_1(\mathcal{V}) \xrightarrow{\partial} Ch_0(\mathcal{V}) \longrightarrow H_0(\mathcal{V}) \longrightarrow 0. \tag{4}$$

The space $H_1(\mathcal{V})$ is easily seen to be trivial. On the other hand, if the representation V is generated by its K_1-invariants, one has a surjective G-equivariant map

$$\begin{array}{rcccl} Ch_0(\mathcal{V}) & \longrightarrow & V & \longrightarrow & 0 \\ [g,v] & \longmapsto & gv & & \end{array} \tag{5}$$

which factorizes through ∂. Saying that V is the 0-homology of its associated coefficient system means that the kernel of (5) is equal to the $\overline{\mathbb{F}}_p[G]$-submodule of $Ch_0(\mathcal{V})$ generated by the elements of the form

$$[1,\varpi v] - [\varpi, v], \quad \text{for } v \in V_{\sigma_1} = V^{I(1)}. \tag{6}$$

2.2. Level 0 homology of the cœfficient system on the tree associated to the pro-p-universal module.— We consider the case where the representation in question is the universal module $C = \overline{\mathbb{F}}_p[I(1)\backslash G]$ and study the associated G-equivariant cœfficient system $\mathcal{C}$.

Theorem 2.1. — *The pro-p-universal module C is the* 0*-homology of the associated cœfficient system on the tree.*

Proof. — As an $\mathcal{H}$-module, C is filtered by the spaces $(C^{K_i})_{i\geq 1}$ and proposition 1.3 says that the natural maps

$$\bigoplus_{g\in(1) \text{ at distance } i} g(C^{K_1}/C^{I(1)}) \xrightarrow{\sim} C^{K_{i+1}}/C^{K_i} \tag{7}$$

are isomorphisms.

Let $F = \sum_g [g, v_g] \in Ch_0(\mathcal{C})$ be a 0-chain. Take the g's in the system of representatives (1).

First suppose that the support of F is contained in the 1-radius disc, $i.e$ that the g's for which $v_g \neq 0$ are at distance ≤ 1 : they belong to the set

$$\{1, g_a^0\varpi,\ a \in \mathbf{P}_1(\mathbb{F}_p)\}.$$

Write

$$F = [1, v] + \sum_{a\in\mathbf{P}_1(\mathbb{F}_p)} [g_a^0\varpi, v_a] = [1, v] + \sum_{a\in\mathbf{P}_1(\mathbb{F}_p)} g_a^0[\varpi, v_a].$$

Recall that the elements v, v_a lie in C^{K_1}. For F to be in the kernel of (5) means that

$$v + \sum_{a\in\mathbf{P}_1(\mathbb{F}_p)} g_a\varpi v_a = 0.$$

In particular, using isomophism (7) for $i = 1$, each v_a belongs to $C^{I(1)}$ which is the $\mathcal{H}$-module generated by the characteristic function of $I(1)$ which we denote by e : set $v_a = h_a e$, with $h_a \in \mathcal{H}$. Then

$$F = [1, -\sum_a h_a g_a^0\varpi e] + \sum_a g_a^0[\varpi, h_a e] = \sum_a g_a^0([\varpi, h_a e] - [1, \varpi h_a e])$$

has the desired form (6).

Suppose now that the support of F is contained in the i-radius disc, with $i \geq 2$. Write

$$F = F_{i-1} + \sum_{\substack{g \text{ at dist. } i-1\\ \text{in the set } (1)}} \sum_{a\in\mathbf{P}_1(\mathbb{F}_p)} [gg_a^0\varpi, v_{g,a}]$$

where F_{i-1} is an element of $Ch_0(\mathcal{C})$ whose support is contained in the $i-1$-radius disc. If F is in the kernel of (5), then $\sum_{a\in\mathbf{P}_1(\mathbb{F}_p)} gg_a\varpi v_{g,a}$ belongs to C^{K_i} and in particular, using isomophism (7), each $v_{g,a}$ belongs to $C^{I(1)}$. This implies that $g_a^0\varpi v_{g,a}$ lies in C^{K_1} so that the following element of $Ch_0(\mathcal{C})$ is well defined, has support in the $i-1$-radius disc, and belongs to the kernel of (5) :

$$F' = F_{i-1} + \sum_g [g, \sum_{a\in\mathbf{P}_1(\mathbb{F}_p)} g_a^0\varpi v_{g,a}].$$

By induction, F' has the form (6). But

$$F - F' = \sum_g \{[g, \sum_{a\in\mathbf{P}_1(\mathbb{F}_p)} g_a^0\varpi v_{g,a}] - \sum_{a\in\mathbb{P}_1(\mathbb{F}_p)} [gg_a^0\varpi, v_{g,a}]\} = \sum_g \sum_{a\in\mathbf{P}_1(\mathbb{F}_p)} gg_a^0\{[1,\varpi v_{g,a}] - [\varpi, v_{g,a}]\}$$

equally has the desired form (6).

□

Remark 2.2. — Proposition 1.7 and theorem 2.1 are actually equivalent !

2.3. Smooth $\overline{\mathbb{F}}_p$-representations of $\mathrm{GL}_2(\mathbb{Q}_p)/p^{\mathbb{Z}}$ and 0-homology of cœfficient systems.— Until mentioned, there still is no restriction on F. The spaces of chains associated to C are endowed with structures of $\mathcal{H}$-modules by induction of the natural left action of $\mathcal{H}$ on C which commutes with the action of G. The exact sequence of G-representations

$$0 \longrightarrow Ch_1(\mathcal{C}) \xrightarrow{\partial} Ch_0(\mathcal{C}) \longrightarrow C \longrightarrow 0. \tag{8}$$

is equally a sequence of $\mathcal{H}$-modules.

Suppose now that the residue field of F is $\mathbb{F}_p$. Then the space C is projective as an $\mathcal{H}$-module ([**O1**]) so that the exact sequence of $\mathcal{H}$-modules is split. Tensoring (8) by a right $\mathcal{H}$-module M then gives an exact sequence compatible with the action of G :

$$0 \longrightarrow ind_{\mathfrak{K}(\sigma_1)}^G(M\otimes_{\mathcal{H}} C^{I(1)})\otimes\epsilon \longrightarrow ind_{\mathfrak{K}(\sigma_0)}^G M\otimes_{\mathcal{H}} C^{K_1} \xrightarrow{\partial} M\otimes_{\mathcal{H}} C \longrightarrow 0. \tag{9}$$

Remark 2.3. — The sequence of G-representations (8) is split by the section

$$\begin{array}{ccc} C & \longrightarrow & Ch_0(\mathcal{C}) \\ e & \longmapsto & [1,e]. \end{array}$$

This map is not $\mathcal{H}$-equivariant map, as we may check by noticing for example that it associates to the element $T_\varpi e = \varpi^{-1}e$ the function with support $K\varpi$ and value e at ϖ, whereas $T_\varpi[1,e]$ is defined to be the function with support K and value $T_\varpi e$ at 1.

Suppose now that F is equal to $\mathbb{Q}_p$. Then the tensor product by the $\mathcal{H}$-module C gives an equivalence between the category of right $\mathcal{H}$-modules and the category of smooth $\overline{\mathbb{F}}_p$-representations of G generated by their $I(1)$-invariants. This functor is a quasi inverse of the functor obtained by taking the space of $I(1)$-invariants of a representation of G [**O2**].

Let V be a smooth $\overline{\mathbb{F}}_p$-representation of G generated by its $I(1)$-invariants. Taking $M = V^{I(1)}$ in the previous discussion gives an exact complex of G-representations

(10)
$$0 \longrightarrow ind^G_{\mathfrak{K}(\sigma_1)} V^{I(1)} \otimes \epsilon \longrightarrow ind^G_{\mathfrak{K}(\sigma_0)} V^{I(1)} \otimes_{\mathcal{H}} C^{K_1} \xrightarrow{\partial} V^{I(1)} \otimes_{\mathcal{H}} C \simeq V \longrightarrow 0.$$

The space $V^{I(1)} \otimes_{\mathcal{H}} C^{K_1}$ injects into V^{K_1} since the $\mathcal{H}$-module C^{K_1} is a direct summand of C [**O1**]. It is therefore a finite dimensional vector space as soon as V is an admissible representation. One gets this way a *standard presentation* for the representation V in the sense of [**C**] : the relevant K-stable subspace generating the G-representation V is the space $V^{I(1)} \otimes_{\mathcal{H}} C^{K_1}$, that is to say, the subspace of V which is generated as a K-representation by $V^{I(1)}$. In the words of Breuil-Paskunas [**B-P**], any smooth $\overline{\mathbb{F}}_p$-representation of G generated by its $I(1)$-invariants is the homology of the diagram

$$V^{I(1)} \hookrightarrow V^{I(1)} \otimes_{\mathcal{H}} C^{K_1}.$$

Remark 2.4. — 1. We know the structure of C^{K_1} as a finitely generated $\mathcal{H}$-module.

2. In the isomophism $V^{I(1)} \otimes_{\mathcal{H}} C \simeq V$, the space $V^{I(1)} \otimes_{\mathcal{H}} C^{K_1}$ doesn't always identify with V^{K_1} (for example for a supersingular representation, see [**B-P**]).

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