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Roll waves on flowing cornstarch suspensions

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Received 1 December 2004; accepted 9 February 2005

Communicated by C.R. Doering

Abstract

We report a traveling wave instability in low Reynolds number flows of aqueous concentrated suspensions of corn starch. The experimental observations are difficult to reconcile with theoretical predictions based on simple rheological models which indicate that flows are stable at low Reynolds number.

1. Introduction

Aqueous suspensions of corn starch show several remarkable features, as many know from observations in the kitchen, or from dining hall trials in English schools; these result from its non-Newtonian rheology. One salient property is an apparent shear thickening: it possesses a resistance to flow that *increases* with the flow rate. For example, a probe, when gently applied, slides easily into the material; however, when rapidly inserted, it encounters considerable resistance, and causes the surface to almost solidify and even fracture. Alternatively, a compressed handful of corn starch suspension appears dry and temporarily solid; it can be crumbled and easily breaks. However, when allowed to relax, it appears to melt and flow away. The time over which this change occurs reflects a relaxation timescale, such as exists for viscoelastic fluids. Nevertheless, the material is not commonly thought of as an archetypal viscoelastic fluid and has never been shown to exhibit some of the classical viscoelastic rheological features, such as the rod climbing of a rotating spindle, open siphoning, or the elastic recoil of a filament.

Although a commonly encountered material, to our knowledge, only two fluid dynamical experiments with cornstarch suspensions have been reported. Most recently, Merkt et al. [1] performed the Faraday experiment with corn starch suspensions, uncovering some novel flow dynamics. In particular, they found that

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0375-9601/$ – see front matter © 2005 Published by Elsevier B.V.
doi:10.1016/j.physleta.2005.02.071

persistent "holes" appear in the vibrated fluid layer, with surrounding elevated collars, which they attribute to the shear-thickening property of the material. Second, Simpson [2] reported the formation of waves on a flowing layer of custard (which is cornstarch with flavouring), meant as a laboratory analogue of a debris flow. Simpson's observations remain unquantified experimentally and unexplained theoretically, and our purpose in the present Letter is to record efforts in this direction.

Instabilities on flowing films of water are an everyday phenomena, being seen on gutters and windows on rainy days, and have a well-established theoretical rationalization in terms of the linear instability of a uniform flow [3,4]. This is the so-called Kapitza problem, for which theory predicts that instabilities arise when the Reynolds number, $Re = UH/\nu$, based on the surface flow speed, U, and depth, H, exceeds a critical value of order unity (ν is the kinematic viscosity). The instability prompts the growth of what are commonly referred to as "roll waves" [5,6], which resemble propagating hydraulic jumps. The surprising property of the corn-starch suspensions reported by Simpson [2], and examined in greater detail herein, is that similar waves arise, but at Reynolds numbers (defined in terms of an effective viscosity) far below the critical value appropriate for a Newtonian fluid. We proceed by reporting the results of an experimental investigation of this phenomenon, indicating how they run counter to current theoretical explanation, and discussing a number of physical mechanisms that might be responsible.

2. Experiments

Experiments were conducted on the flow down a constant incline of a concentrated solution of corn starch in water. We used a number of brands of commercially available cornstarch (e.g., Arco, Safeways). The laboratory set-up involved a rectangular chute, 10 cm wide and 1.5 or more meters long, fed upstream from a reservoir whose level was kept roughly constant by continually adding fluid. By changing the angle of inclination, we were able to vary flow speed and depth. Typical fluid thickness were of the order of 0.5–1 cm; flow speeds were in the 1–10 cm/s range. We also varied the concentration of the cornstarch in solution; suspensions slightly in excess of 1 part cornstarch to 1 part water, by weight, were those marked by the most pronounced surface waves.

If the fluid were Newtonian, the force of gravity down the plane, associated with the gravitational acceleration $g\sin\theta$ (where θ is the angle of the plane and $g \approx 9.81\ \mathrm{m/s^2}$), would be balanced by that stemming from the viscous shear stress, μu_{zz} (μ being the viscosity, and orientating a two-dimensional Cartesian coordinate system so that $z = 0$ denotes the inclined plane, x points directly downslope, and (u, w) denote the velocity field). This demands that

$$\frac{\nu U}{H^2} \sim g\sin\theta;$$

hence, $Re = UH/\nu \sim U^2/(gH\sin\theta)$. For the roll wave typically seen on water films, the fluid is millimeters in depth and flows at centimeters per second on a shallow slope of perhaps 10 degrees. This indicates a Reynolds number of order 10 or more, which is comfortably above the critical value of $(5/4)\cot\theta$ [3,4] anticipated for the Kapitza problem. For our cornstarch solutions, on the other hand, assuming that the flow is controlled by an equivalent viscous shear stress, we observe waves at effective Reynolds numbers of order 0.1 or less.

Photographs of the roll waves are shown in Fig. 1(a) and (b). Naturally arising perturbations at the inlet seed growing, propagating disturbances that steepen

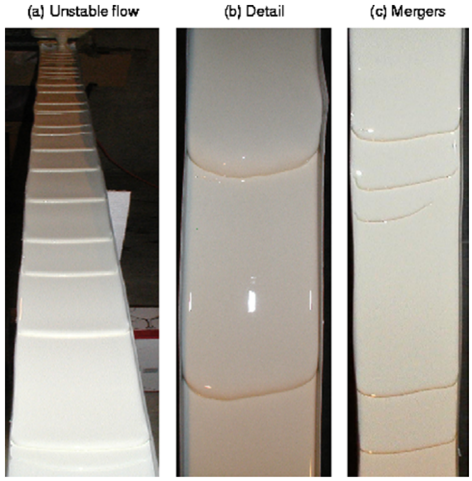

Fig. 1. A typical experimental view of (a) developed roll-waves, (b) a detailed view of a pair of waves, and (c) merging waves at a downstream location; to give an idea of scale, the width of the chute is 10 cm.

into an unsteady train of waves. Near initiation, the waves are remarkably regular, and fairly evenly spaced, with a wavelength of a few centimeters. They quickly grow and reach relatively large amplitudes; the heights of their crests can be a significant fraction of the fluid depth. There is a range of crest heights and as a result, the waves travel with different speeds, overtaking one another in their progression down the chute. The collision of two waves leads to merger into a larger wave. Thus the wavetrain undergoes a process of coarsening, as illustrated in Fig. 1(c), which is typical of many non-linear wave propagation problems (e.g., [7]). Both the initial wavelengths and the coarsened wavelengths further down the channel showed little dependence on the chute inclination. In most of the flows, there was little cross-stream variation of the roll-wave profiles (except near the side walls), and what remaining structure emerged appeared to stem from perturbations at the inlet (but see the remarks below regarding aged material). However, it could well be possible that the roll wave develop transverse variations through secondary instabilities in wider channels.

Though a range was observed, wave speeds typically exceeded the mean flow speed by approximately 33–50 percent. The structure of the waves is also distinctive: the fluid surface appears to be advected with the waves, creating a characteristic caterpillar action as the disturbances “roll” over the fluid layer ahead. This is particularly evident at the front of the current, which progresses through a series of surges.

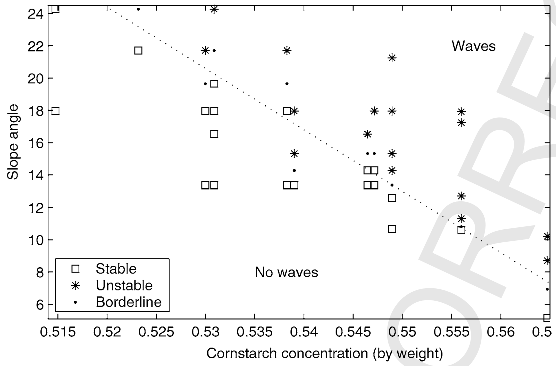

Fig. 2. Flows with and without unstable roll waves, plotted on a graph of slope angle against concentration (fraction of cornstarch by weight). The stars show flows in which waves appeared to amplify as they propagated downstream, the squares represent flows in which the waves appeared to decay, and the dots show cases that seemed borderline between those two behaviours. The lines are a fit to the data.

Lastly, by varying the inclination, we observe what appears to be a critical threshold in slope below which small waves initiated at the top of the chute no longer grow or persist as they propagate, but instead decay. The dependence of critical angle on concentration is presented in Fig. 2. The critical angle is difficult to establish for two reasons. First, and less importantly, the chute is not sufficiently long to state definitively whether a small disturbance grows or decays over the duration of the experiment. Second, even though it is clear that small amplitude waves decay in the “stable” regime in the experiment, it appears that larger amplitude disturbances are able to persist and survive. In other words, finite-amplitude effects significantly complicate the identification of the threshold slope.

3. Other observations

Jitter In addition to the main wave generation process, the flow of the corn starch suspension exhibited another curious feature resembling a superimposed high-frequency jitter or flutter. More precisely, the large-scale flow of the material appeared to generate a high-frequency vibration in a manner reminiscent of the generation of acoustic waves by flow in a compressible fluid. The jitter could be seen clearly at the crests of the roll waves, where the caterpillar-like overturning motions vibrated with periods of order 0.1 seconds. However, simply pouring the material from one receptacle to another also excited the vibrations, to the degree that they could be easily felt on holding one of the containers. This process was also presumably responsible for creating substantial agitation at the inlet, which in turn seeded the roll waves themselves.

Ageing After repeating experiments over the course of several days it became clear that the properties of the corn starch suspensions were slowly evolving, presumably owing to some combination of evaporation of the suspending fluid and the swelling of the starch grains. For example, the material used in the experiment illustrated in Fig. 1 was fresh (prepared an hour before the experiment). Leaving the suspension for a period of a day or two led to partial separation be-

Fig. 3. Roll waves on fluid layers of aged material. The second panel shows an experiment in which the inclination was steeper than that shown in the first panel.

tween the cornstarch and suspending fluid; however, remixing produced a suspension that appeared much the same as before. When the flow experiments were repeated at this time, the roll waves that appeared were markedly different: the critical slope angle increased, the roll-wavelength was visibly smaller, the instability appeared to saturate at much lower amplitude and there was little sign of coarsening. Snapshots of the re-run experiment are shown in Fig. 3. Tranverse variations in the roll-wave profiles were also evident in several of the flows of aged material, with wave patterns propagating across the stream even in our relatively narrow channel. We note that solutions of one particular brand of cornstarch (Challenge-ACH food companies, inc.) behaved more like the aged suspensions of the other brands.

4. Theoretical background

The main theoretical problem posed by the cornstarch suspension is characterizing its rheology. Existing studies suggest that the suspension can be shear thickening (and even thinning over some ranges of applied shear) [1] and show a definite relaxation time, much as we anticipated on observational grounds. However, there is currently no accepted rheological model for this material. Instead, in the absence of such a model, we review known theoretical results for commonly used models of shear thickening or visco-elastic fluids.

Shear-thickening behaviour is captured in the standard power-law fluid model, for which the viscosity, μ, depends on the local deformation rate, $\dot{\gamma}$: $\nu = K\dot{\gamma}^{n-1}$, where n and K are constants ($n > 1$ implies shear thickening). Long-wave stability theory [8] establishes that the critical Reynolds number for this flow, $Re = U^{n-2}H^n/K$, is given by $Re = Re_c \equiv (2n+3)/(4\tan\theta)$. (For $n = 1$ we recover the familiar Newtonian result of Benjamin [3] and Yih [4].) The critical threshold increases with n, and so shear thickening fluids (with $n > 1$) are *more* stable than Newtonian and shear thinning ones. Thus, the observed behaviour of the cornstarch suspension cannot be rationalized in terms of shear-thickening rheology alone.

Roll waves have been observed on mud flows (kaolin suspensions) in laboratory flumes [8,9], and mud is usually thought to be a shear-thinning viscoplastic fluid. The experimental observations suggest critical Reynolds numbers that are much higher than those observed for our cornstarch suspensions, and the observed wavelengths of muddy roll waves are much longer than those seen in the cornstarch suspensions. Unlike cornstarch, the observations for mud are consistent with theoretical predictions based on standard rheological models for viscoplastic fluids [10]. Interestingly, the theory applied to shear thickening, viscoplastic fluids [10] suggests that yield stresses could substantially lower the critical Reynolds number. However we observed that cornstarch suspensions show negligible yield stresses, draining from inclined surfaces over long timescales to very thin films.

Long-wavelength stability theories have been presented for a number of archetypal viscoelastic fluids [11,12]. In particular, for the Oldroyd-B model, it has been shown that elasticity can be destabilizing: the critical Reynolds number is lowered by an amount proportional to the dimensionless polymer relaxation rate, the Deborah number. If the relaxation and shear rates are comparable, the dynamics is likely to be strongly influenced by the elasticity. In our experiments, both the observed relaxation rate (inferred roughly from the time taken for compressed, rigid, material to return to a fluid state after compression is released) and shear rate are of order one Hertz. The possibility thus exists that the viscoelastic behaviour of the material is responsible for the diminution of the critical Reynolds number. However, as previously mentioned, the cornstarch suspension is not usually thought of in the same vein as polymeric, viscoelastic fluids. Moreover, it has been questioned previously whether this type of viscoelastic instability could ever be observed [12].

Roll waves have also been reported on flowing granular layers [13]. The theory used to rationalize this phenomenon is based on shallow-fluid models akin to the St. Venant model of hydraulic engineering. Simple friction laws model the granular fluid stresses, and yield some agreement between theory and observation. However, the flow speeds required for wave formation are well above those encountered in our cornstarch suspensions, and more in line with those encountered for Newtonian viscous films.

5. Discussion

In summary, we are unable to rationalize the appearance of roll waves at low Reynolds number in cornstarch suspensions by adopting a simple rheological model. To provide an answer to this puzzle, more studies are required on this material, both at the microscopic and macroscopic levels, in order to characterize its rheology. Because we have no convincing theoretical explanation, we can only speculate as to possible mechanisms.

Most obviously, the cornstarch suspension could be visco-elastic, and the known stability results for Oldroyd-B fluids [12] might hold the key to the puzzle. However, Oldroyd-B fluids are not shear thickening. Nevertheless, more general versions of that constitutive law (such as the Oldroyd-8 family) can incorporate shear thickening in simple steady shear and extensional flow [14]. The question then arises as to whether one can fit rheological measurements of cornstarch suspensions to such a model.

Alternatively, wave excitation at low Reynolds numbers could be attributed to a jamming phenomenon as may arise in highly concentrated suspensions [15,16]: a localized perturbation may jam particles together into a coherent structure that accelerates to collect and jam further particles into a growing mass. Related notions have appeared in other contexts, such as in thixotropic fluids where it has been suggested that particle interaction can lead to a viscosity bifurcation and thence instability [17,18]. In a sense, this behaviour results from what might be called “constitutive instability”, which is also a common concept in non-Newtonian fluid mechanics, primarily for viscoelastic fluids [19,20].

Another possibility is related to the idea that the suspension develops inhomogeneity: suspended particles are known to migrate in solute, leaving regions of high shear or boundary layers [21]. Effective slip in the particle-depleted layers, both internal and at a wall, can result from this (a phenomenon that plagues rheologists). A similar dynamic sorting arises in media like wet sand, where shear perturbs the solid matrix from its state of optimal packing, thus increasing the fluid fraction at the base of the layer. In both cases, a more viscous, particle-rich layer develops over a less viscous layer. A stick-slip instability is then a possible consequence, as is interfacial instability due to the abrupt change in effective viscosity [22]. Although such explanations are appealing, it remains unclear whether the form of the resulting instability could resemble roll waves.

For the observed high frequency jitter, could there be a link to three other phenomena? First, the flow of compressible fluids can generate acoustic waves (e.g., [23]). Second, elasto-plastic materials suffer what is referred to as “flutter” instability [24], which takes the form of unstable propagating waves with speeds near those expected for shear and compressional waves. Third, the avalanching of so-called singing or booming sand and fluidized granular media may produce acoustic signals [25,26]. In all three instances, a relatively slow flow generates high frequency wave-like disturbances.

Whatever the physical origin of the observed phenomenon, we hope that our study will motivate further investigations of this curious material.

Acknowledgements

We thank Anthony Pierce, Bud Homsy, Omar Matar and Gareth McKinley for helpful discussions, and John Simpson for drawing our attention to this problem.

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