Marks

[42] **1. Short-Answer Questions.** Put your answer in the box provided but show your work also. Each question is worth 3 marks, but not all questions are of equal difficulty. Full marks will be given for correct answers placed in the box, but at most 1 mark will be given for incorrect answers. Unless otherwise stated, it is not necessary to simplify your answers in this question.

(a) Evaluate $\lim_{h\to 0} \dfrac{(3+h)^2 - 9}{h}$ or determine that this limit does not exist.

Answer

(b) Evaluate $\lim_{x\to\infty} (\sqrt{4x^2 + x} - 2x)$ or determine that this limit does not exist.

Answer

(c) Find all values of the constant c that make the function f continuous everywhere, or determine that no such value exists:

$$f(x) = \begin{cases} \dfrac{\sin(4x)}{x} & \text{if } x \neq 0, \\ c & \text{if } x = 0 \end{cases}$$

Answer

Continued on page 3

(d) Find the derivative of $(t^3 + 2t)e^t$.

Answer

(e) Find the derivative of $y = \dfrac{\sin x}{x^4}$.

Answer

(f) Find $f'(x)$, if $f(x) = e^{\cos x}$.

Answer

(g) Find the slope of the tangent line to the curve $\sqrt{x} + 3\sqrt{y} = 5$ at the point $(4, 1)$.

Answer

Continued on page 4

(h) Find y' if $y = \sin^{-1}(x^3)$. [*Note:* Another notation for $\sin^{-1}$ is arcsin.]

Answer

(i) Find $f'(x)$ if $f(x) = x^{\sin x}$.

Answer

(j) Use a linear approximation to estimate $(1.999)^4$.

Answer

(k) Find the first three nonzero terms in the Maclaurin series for $f(x) = x^4 \sin(x^2)$.

Answer

Continued on page 5

(l) Find the absolute maximum *value* of $f(x) = x^{2/3}$ on the interval $[-1, 2]$.

Answer

(m) Newton's Method is used to approximate a solution of the equation $x + \ln x = 0$, starting with the initial approximation $x_1 = 1$. Find x_2.

Answer

(n) A particle is moving with velocity function $v(t) = \cos t - \sin t$ and initial displacement $s(0) = 0$. Find the displacement at any time t.

Answer

Continued on page 6

Full-Solution Problems. In questions 2–6, justify your answers and **show all your work**. If a box is provided, write your final answer there. Simplification of answers is not required unless explicitly requested.

[10] **2.** A bacteria culture grows with constant relative growth rate. After 2 days there are 40,000 bacteria and after 7 days the count is 4 billion $= 4 \cdot 10^9$.

(a) Write a differential equation satisfied by the bacteria population at any time t.

Answer

(b) Find the initial population, expressed as an integer.

Answer

(c) Find the population after t days.

Answer

Continued on page 7

[10] **3.** An airplane flies horizontally at an altitude of 6 km and passes directly over a tracking telescope on the ground. When the angle of elevation (i.e. the angle at the telescope measured upwards from the horizontal to the airplane) is $\pi/6$, this angle is decreasing at a rate of 40 rad/min. How fast is the airplane travelling at that time?

Answer

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[12] **4.** Let $f(x) = x^{5/3} + \frac{5}{2}x^{2/3}$.

(a) (1 mark) Find the domain of $f(x)$.

(b) (4 marks) Determine intervals where $f(x)$ is increasing or decreasing and the x- and y-coordinates of all local maxima or minima (if any).

(c) (3 marks) Determine intervals where $f(x)$ is concave upwards or downwards, and the x-coordinates of all inflection points (if any).

Question 4 continues on the next page...

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Question 4 continued

(d) (2 marks) Find and verify the equations of any asymptotes (horizontal, vertical or slant), or else determine that there are no asymptotes.

(e) (4 marks) Sketch the graph of $y = f(x)$, showing the features given in items (a) to (d) above and giving the (x, y) coordinates for all points occurring above and also all x-intercepts (if any).

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[10] **5.** Find (with justification) the dimensions of the rectangle of largest area that has its base on the x-axis and its other two vertices above the x-axis and lying on the parabola $y = 15 - x^2$.

Answer

Continued on page 11

[4] **6.** Use the *definition of the derivative* to find $f'(x)$, if

$$f(x) = \sqrt{x+1}.$$

You may not use derivative formulas such as the Power Rule or the Chain Rule to answer this question.

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[4] **7.** Determine what degree Maclaurin polynomial for $\ln(1-x)$ that should be used to approximate $\ln(1.1)$, so that the approximation is guaranteed to be accurate to within 10^{-9}.

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[8] **8.**

(a) (4 marks) Prove that $x + \ln|x| = 0$ has at least one solution in the open interval $(-1, 1)$.

(b) (4 marks) Prove that $x + \ln|x| = 0$ has exactly one solution in the open interval $(-1, 1)$.

The End

Be sure that this examination has 13 pages including this cover

The University of British Columbia

Sessional Examinations - December 2009

Mathematics 100/180

Differential Calculus with Applications to Physical Sciences and Engineering

Closed book examination Time: 2.5 hours

Surname(s): _____ **Given Name(s):** _____

Student Number: _____ **Instructor's Name:** _____

Signature: _____ **Section Number:** _____

Rules governing examinations

1. Each candidate should be prepared to produce his or her library/AMS card upon request.
2. Read and observe the following rules:

No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination.

Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.

CAUTION - Candidates guilty of any of the following or similar practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

(a) Making use of any books, papers or memoranda, other than those authorized by the examiners.

(b) Speaking or communicating with other candidates.

(c) Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

3. Smoking is not permitted during examinations.

1		42
2		10
3		10
4		12
5		10
6		4
7		4
8		8
Total		100