Solution guaranteed

Consider the differential equation

$$(x-2)y'' + y' + (x-2)(\tan x)y = 0, \qquad y(3) = 1,\ y'(3) = 2.$$

Without solving the equation, find the longest interval in which this initial value problem is certain to have a unique solution.

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A. $(\frac{\pi}{2}, 2)$.

B. $(2, 3)$.

C. $(2, \frac{3\pi}{2})$.

D. $(2, 3]$.

E. $(\frac{3\pi}{2}, \frac{5\pi}{2})$.

The principle of superposition

For which of the following equations does the principle apply?

A. $yy'' + y'^2 = 0$.

B. $y'' + y' - 2y = \sin t$.

C. $ty'' + 3y = t$.

D. $yy' = e^t$.

E. $t^2y'' - t(t+2)y' + (t+2)y = 0$.

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A. $te^{-t} + Ct$

B. $te^{t} + Ct$

C. $e^{t} + Ct$

D. $(t + C)^2e^{t}$

E. $(t + C)^2e^{-t}$

Here C denotes an arbitrary constant.

Fundamental solutions

Consider the equation $y'' + p(t)y' + q(t)y = 0$ on an open interval I, where the coefficients $p(t)$ and $q(t)$ are continuous and nonvanishing everywhere I. Determine which of the following statements is false.

If $\{y_1, y_2\}$ is a fundamental set of solutions of this equation, then

A. y_1 and y_2 cannot have a common zero on I.

B. y_1 and y_2 cannot have a common maximum or minimum point on I.

C. y_1 and y_2 cannot have a common inflection point (where the second derivative vanishes) on I.

D. Every solution of the ODE on I is of the form $y = c_1y_1 + c_2y_2$ where c_1 and c_2 are constants.

E. There is a constant c such that $y_2 = cy_1$ on I.

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Prove the statements that are true.