Math 421/510 Quiz 5 Solution

1. Is it possible to define an inner product on $C[0,1]$ which induces the sup norm? Recall that the sup norm is given by

$$||f||_\infty = \sup_{x\in[0,1]} |f(x)|.$$

(10 points)

Solution. If the sup norm was indeed generated by an inner product, then it would have to obey the parallelogram law:

$$||f_1 + f_2||_\infty^2 + ||f_1 - f_2||_\infty^2 = 2\big(||f_1||_\infty^2 + ||f_2||_\infty^2\big).$$

It is easy to check that this identity is fase; try for example $f_1 \equiv 1,\ f_2(x) = x$. $\square$

Here is a related question: *does there exist an inner product on* $C[0,1]$ *there generates the same topology as that of the sup norm?* Note that the above solution does not suffice in this case.