1. $$\frac{1}{j^2} - \frac{1}{(j+1)^2} = \frac{j^2 + 2j + 1 - j^2}{j^2(j+1)^2} = \frac{2j+1}{j^2(j+1)^2}$$

$$\sum_{j=1}^{n} \frac{2j+1}{j^2(j+1)^2} = \sum_{j=1}^{n} \left(\frac{1}{j^2} - \frac{1}{(j+1)^2} \right)$$

$$= \frac{1}{1^2} - \frac{1}{(n+1)^2} = \frac{n^2 + 2n}{(n+1)^2}$$

14. $g(\theta) = \int_{e^{\sin\theta}}^{e^{\cos\theta}} \ln x \, dx$

$$g'(\theta) = (\ln(e^{\cos\theta}))e^{\cos\theta}(-\sin\theta) - (\ln(e^{\sin\theta}))e^{\sin\theta}\cos\theta$$

$$= -\sin\theta\cos\theta(e^{\cos\theta} + e^{\sin\theta})$$

1. $x_i = 2^{i/n}$, $0 \le i \le n$, $f(x) = 1/x$ on $[1, 2]$. Since f is decreasing, f is largest at the left endpoint and smallest at the right endpoint of any interval $[2^{(i-1)/n}, 2^{i/n}]$ of the partition. Thus

$$\begin{aligned} U(f, P_n) &= \sum_{i=1}^{n} \frac{1}{2^{(i-1)/n}} (2^{i/n} - 2^{(i-1)/n}) \\ &= \sum_{i=1}^{n} (2^{1/n} - 1) = n(2^{1/n} - 1) \\ L(f, P_n) &= \sum_{i=1}^{n} \frac{1}{2^{i/n}} (2^{i/n} - 2^{(i-1)/n}) \\ &= \sum_{i=1}^{n} (1 - 2^{-1/n}) = n(1 - 2^{-1/n}) = \frac{U(f, P_n)}{2^{1/n}}. \end{aligned}$$

Now, by l'Hôpital's rule,

$$\begin{aligned} \lim_{n\to\infty} n(2^{1/n} - 1) &= \lim_{x\to\infty} \frac{2^{1/x} - 1}{1/x} \qquad \left[\frac{0}{0}\right] \\ &= \lim_{x\to\infty} \frac{2^{1/x} \ln 2(-1/x^2)}{-1/x^2} = \ln 2. \end{aligned}$$

Thus $\lim_{n\to\infty} U(f, P_n) = \lim_{n\to\infty} L(f, P_n) = \ln s$.

68. $\displaystyle\int \frac{x\,dx}{4x^4+4x^2+5}$ Let $u = x^2$
$du = 2x\,dx$

$$= \frac{1}{2}\int \frac{du}{4u^2+4u+5}$$

$$= \frac{1}{2}\int \frac{du}{(2u+1)^2+4}$$ Let $w = 2u+1$
$dw = 2du$

$$= \frac{1}{4}\int \frac{dw}{w^2+4} = \frac{1}{8}\tan^{-1}\left(\frac{w}{2}\right) + C$$

$$= \frac{1}{8}\tan^{-1}\left(x^2+\frac{1}{2}\right) + C.$$