Math 121 Assignment 3

Due Friday January 29

1. Evaluate the following integrals:

$(a) \displaystyle\int \frac{\ln(\ln x)}{x}\,dx$ $\quad (b) \displaystyle\int (\arcsin(x))^2\,dx$ $\quad (c) \displaystyle\int xe^x \cos x\,dx$

$(d) \displaystyle\int \frac{x^3+1}{12+7x+x^2}\,dx$ $\quad (e) \displaystyle\int \frac{dt}{(t-1)(t^2-1)^2}$ $\quad (f) \displaystyle\int \frac{dx}{e^{2x}-4e^x+4}$

$(g) \displaystyle\int \frac{dx}{x^2(x^2-1)^{\frac{3}{2}}}$ $\quad (h) \displaystyle\int \frac{dx}{x^2(x^2+1)^{\frac{3}{2}}}$ $\quad (i) \displaystyle\int \frac{d\theta}{1+\cos\theta+\sin\theta}.$

2. Use the method of undetermined coefficients to evaluate the integral $\int x^2(\ln x)^4\,dx$.

3. Write down the form that the partial fraction expansion of

$$\frac{x^5+x^3+1}{(x-1)(x^2-1)(x^3-1)}$$

will take. Do not evaluate the constants.

4. Consider the integral $I = \int e^{-x^2}\,dx$. It is known (and you can use this fact without proof) that if it were possible to evaluate the integral I using elementary functions (these are functions that can be written as compositions of functions that are polynomial, trigonometric or exponential, or their inverses), it would take the form

$$I = P(x)e^{-x^2} + C$$

where P is a polynomial. Show that such a polynomial P does not exist. This is called a proof by contradiction, and it shows that an elementary function (such as e^{-x^2}) may very well possess non-elementary anti-derivatives.

5. Obtain reduction formulae for

$$I_n = \int_0^{\frac{\pi}{2}} x^n \sin x\,dx \qquad \text{and} \qquad J_n = \int \sin^n x\,dx$$

and use them to evaluate I_6 and J_7.