Math 121 Assignment 1

Due at the beginning of class on Friday January 15

■ Instructions:

- Please staple all relevant pages of your work. Make sure your name and student ID are included at the top.
- Submitted work should be clean, legible and written in complete English sentences. A correct answer without adequate justification of the intermediate steps will receive no credit.
- Late homework will not be accepted without prior consent of the instructor.

1. (a) Write the sum

$$2^2 - 3^2 + 4^2 - 5^2 + \cdots - 99^2$$

using the sigma notation. Then evaluate it.

(b) Repeat the same exercise as above for the sum

$$\frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{4}{16} + \cdots + \text{upto the first } n \text{ terms.}$$

2. Use mathematical induction to show that

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}.$$

3. Use Riemann sums to compute accurately (not estimate) the areas of the regions specified below:
(a) Below $y = x^2 + 2x + 3$, above $y = 0$, from $x = -1$ to $x = 2$.
(b) Above $x^2 - 2x$, below $y = 0$.

4. (a) If P_1 and P_2 are two partitions of $[a, b]$ such that every point of P_1 also belongs to P_2, then we say that P_2 is a refinement of P_1. Show that in this case

$$L(f, P_1) \leq L(f, P_2) \leq U(f, P_2) \leq U(f, P_1).$$

(b) Use the result above to show that every lower Riemann sum is less than or equal to every upper Riemann sum. This fact was critical in our defintion of the Riemann integral.

5. Using the interpretation of Riemann integral as area and using properties of the definite integral, evaluate:

(*a*) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2n+3i}{n^2}$.

(*b*) $\int_{-3}^{3} (2+t)\sqrt{9-t^2}\, dt$.

(*c*) $\int_{-1}^{2} \operatorname{sgn}(x)\, dx$, where $\operatorname{sgn}(x) = \begin{cases} +1 & \text{if } x > 0, \\ -1 & \text{if } x < 0. \end{cases}$

(*d*) $\int_{-3}^{4} \left(|x+1| - |x-1| + |x+2|\right)\, dx$.

(*e*) $\int_{0}^{3} \frac{x^2 - x}{|x-1|}\, dx$.

(*f*) the constant k minimizing the integral $\int_{a}^{b} \left(f(x) - k\right)^2\, dx$, where f is a continuous function on $[a, b]$, $a < b$.