Review worksheet for Midterm 1

Math 300, Section 202, Spring 2015

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1. Find the limit of the function $f(z) = (z/\bar{z})^2$, if it exists, as z tends to zero. If you think the limit does not exist, explain your reasoning for this conclusion.

   (*Answer: The limit does not exist.*)

2. Describe geometrically the collection of points z satisfying the equation $|z-1| = |z+i|$. Sketch this set of points in the complex plane.

   (*Answer: A straight line through the origin with slope* -1.)

3. Express the complex number $(-1+i)^7$ in the form $a+ib$.

   (*Answer:* $-8(1+i)$)

4. Decide whether the set $\{z : 0 \leq \arg(z) \leq \frac{\pi}{4}\}$ is bounded. Give reasons for your answer.

   (*Answer: not bounded.*)

5. Describe the domain of definition of the function $f(z) = z/(z+\bar{z})$.

   (*Answer:* $Re(z) \neq 0$.)

6. Find and sketch the images of the hyperbolas

$$x^2 - y^2 = -1 \qquad \text{and} \qquad xy = -2$$

   under the transformation $w = z^2 = (x+iy)^2$.

   (*Answer: The vertical line* $x = -1$ *and the horizontal line* $y = -4$ *respectively.*)

7. Show that the function $f(z) = x^2 + iy^2$ is differentiable at the origin but analytic nowhere.

8. Find the harmonic conjugate of the function $u(x,y) = y^3 - 3x^2y$ if it exists. If the answer is yes, determine the analytic function f whose real part is u.

   (*Answer:* $v(x,y) = -3xy^2 + x^3 + C,\ f(z) = i(z^3 + C)$.)

9. State whether each of the following statements is true or false. If the statement is true, give a short proof of it. If not, give a counterexample to show that it is false.

   (a) The function $f(z) = e^z$ is harmonic.

   (b) $|(2\bar{z}+5)(\sqrt{2}-i)| = \sqrt{3}|2z+5|$

(c) There exists a complex number z_0 whose fourth roots z_1, z_2, z_3, z_4 have the property that

$$\arg(z_1) = \frac{\pi}{4}, \quad \arg(z_2) = \frac{\pi}{2}, \quad \arg(z_3) = \frac{2\pi}{3}, \quad \arg(z_4) = \pi.$$

(d) The equation $(z^2 + z + 1)e^z = 0$ has exactly two complex roots.

(e) If a rational function R has a pole at the point a, then then residue of R at a must be a nonzero complex number.

(Answer: (a) True (b) True (c) False (d) True (e) False)