Math 105 Course Outline

Week 8

Overview

Over the past few weeks, we have covered numerous techniques for evaluating definite and indefinite integrals in various forms, but these techniques cannot be used on every integral. In these cases we can use *numerical integration* to approximate the integral and estimate the error in the approximation.

The techniques we have covered thus far all involve finite intervals of integration and finite-valued functions. We can, in some cases, evaluate integrals where these conditions are not met, which are called *improper integrals*.

We can apply our understanding of integration to the field of *differential equations*, which relate an unknown function to its derivatives. They can be used to model many different situations.

Learning Objectives

These should be considered a minimum, rather than a comprehensive, set of objectives.

By the end of the week, having participated in lectures, worked through the indicated sections of the textbook and other resources, and done the suggested problems, you should be able to independently achieve all of the objectives listed below.

Ref	Learning Objective
08–01	Numerical Integration

Objective 1: Given an approximation c and an exact solution x to a problem, compute the absolute and relative error. [Procedural]

Example problem: *Compute the absolute and relative error in using* 0.1 *to estimate* $\frac{1}{9}$.

Reading: Text §7.6 (p. 492)

Practice problems: Text p. 499: *7 – 10.*

Objective 2: Give the definition of Simpson's rule to approximate $\int_a^b f(x)\,\mathrm{d}x$. [Recall]

Objective 3: Given an even integer n, compute an approximation to $\int_a^b f(x)\,\mathrm{d}x$ using Simpson's rule. [Procedural]

While we will not be covering the Midpoint rule or Trapezoidal rule, they are useful in building up your understanding of numerical integration in preparation for Simpson's rule.

Example problem: *Approximate* $\int_2^{10} 2x^4\,\mathrm{d}x$ *with* $n = 4$ *subintervals and Simpson's Rule.*

Reading: Text §7.6 (pp. 492 – 497)

Practice problems: Text p. 499: *35 – 38.*

Objective 4: Calculate and interpret the error bound for an approximation to $\int_a^b f(x)\,\mathrm{d}x$ using Simpson's rule. [Procedural]

You do not need to memorize the formula given in Theorem 7.2.

Example problem: *Find a bound on the error made when approximating* $\int_2^{10} 2x^4\,\mathrm{d}x$ *with* $n = 4$ *subintervals and Simpson's Rule.*

Reading: Text §7.6 (p. 499)

Practice problems: Text p. 499: *44-47.*

Ref	Learning Objective
08–02	Improper Integrals

Objective 1: Recognize and evaluate improper integrals over infinite intervals using the definitions. [Procedural]

The material on solids of revolution will not be covered.

Example problem: *Compute* $\int_0^\infty e^{-3x}\,\mathrm{d}x$.

Reading: Text §7.7 (pp. 502 – 505)

Practice problems: Text p. 510: *5 – 20.*

Objective 2: Recognize and evaluate improper integrals with unbounded integrands using the definitions. [Procedural]

Example problem: *Compute* $\int_0^1 \frac{1}{\sqrt{x}}\,\mathrm{d}x$.

Reading: Text §7.7 (pp. 506 – 509)

Practice problems: Text p. 510: *27 – 36.*

08–03 Differential Equations

Objective 1: Describe the concept of a *differential equation*, including the terms *order* and *general solution*. Describe an *initial value problem*.[Conceptual]

Reading: Text §7.8 (p. 514)

Objective 2: Find the solution to a separable first order differential equation, with or without an initial condition. [Procedural]

Note that differential equations of the form $y'(t) = f(t)$ are separable, as are the first order linear differential equations with constant coefficients covered in the textbook. We will not be covering sketching or interpreting direction fields.

Example problems: *Solve* $y'(t) = 10e^{-t/2}$ *with initial condition* $y(0) = 4$, and *solve* $y'(x) = y^2e^{-x}$ *with initial condition* $y(0) = 1/2$.

Reading: Text §7.8 (pp. 514 – 517)

Practice problems: Text p. 520: *9 – 20, 23 – 32.*

Objective 3: From a word problem, model the problem using a differential equation, then use it to solve the problem. [Procedural]

Practice problems: Text p. 522: *53 – 59.*