Math 105 Course Outline

Week 6

Overview

This week will be about the *Fundamental Theorem of Calculus* that connects differential calculus with integration. The key notion is the definition of an *antiderivative* of a function. We use this result to evaluate definite integrals of functions f that cannot be evaluated with Riemann sums that easily.

We will introduce the *substitution rule* and *integration by parts* as the two important integration techniques to evaluate or simplify definite integrals. For instance, they can be applied to prove that the circle of radius r has area $r^2\pi$.

Learning Objectives

These should be considered a minimum, rather than a comprehensive, set of objectives.

By the end of the week, having participated in lectures, worked through the indicated sections of the textbook and other resources, and done the suggested problems, you should be able to independently achieve all of the objectives listed below.

Ref	Learning Objective
06–01	The Area Function of a Function f

Objective 1: Give the definition of the area function $A(x)$ for a function $f(t)$ with left endpoint a. [Recall]

Objective 2: Given a function $f(t)$ compute the area function using Riemann sums or geometric interpretation and facts about areas. [Procedural]

Example problem: *Let $f(t) = 2t - 3$. Compute the area function $A(x)$ of $f(t)$ with left endpoint $a = -1$.*

Reading: Text §5.3, pp. 334 – 337

Practice problems: Text §5.3, p. 818: 12, 17, 18

06–02	The Fundamental Theorem of Calculus

Objective 1: Give the definition of an antiderivative of a function f. [Recall]

Reading: Text §4.8, pp. 292 – 293

Objective 2: Given a function $f(x)$ find an antiderivative [Procedural]

Practice problems: Text §4.8, p. 301: 11, 13, 17

Objective 3: Give the definition of an indefinite integral of a function f. [Recall]

Objective 3: State the power, constant and sum rule for indefinite integrals. Write down the indefinite integrals of trigonometric functions such as $\sin(ax)$, $\cos(ax)$, $\tan(ax)$, $\sec(ax)$ and apply these to obtain antiderivatives/indefinite integrals for combinations of such functions. [Procedural]

Reading: Text §4.8, pp. 294 – 295d

Practice problems: Text §4.8, p. 301: 19, 22, 25, 27, 28

> **Objective 4**: Explain the arbitrary constant involved in the computation of an indefinite integral. [Conceptual]

> **Objective 5**: Explain the distinction between a definite and indefinite integral with the same integrand. [Conceptual]

> **Objective 6**: Recognise that the area function $A(x)$ with left endpoint a of a function $f(t)$ satisfies $A'(x) = f(x)$. [Conceptual]

Reading: Text §5.3, pp. 337 – 338

Practice problems: Text §5.3, p. 818: 19, 22

> **Objective 7**: State the Fundamental Theorem of Calculus. Use the fundamental theorem of calculus to connect a definite integral with the indefinite integral/antiderivative of the integrand. [Conceptual]

Reading: Text §5.3, Theorem 5.3 Part 1 and 2 (pp. 338 – 339)

> **Objective 8**: Apply the Fundamental Theorem of Calculus to evaluate definite integrals and compute derivatives of integrals considered as functions in the lower or upper bound. [Procedural]

Reading: Text §5.3, pp. 339 – 342

Example problem: *Evaluate the integral $\int_0^{\pi/2} \cos(x)\, dx$ with the help of the fundamental theorem of calculus.*

Practice problems: Text §5.3, pp. 346 – 347: 25, 30, 32, 37, 39, 51-57

06–03   Integration Techniques

> **Objective 1**: State the substitution rule for (in)definite integrals. [Recall]

Reading: Text §5.5, Theorem 5.7 (pp. 361–363)

> **Objective 2**: Apply the substitution rule to transform a given (in)definite integral to a more simple (in)definite integral. [Procedural]

Example problem: *Simplify and then evaluate the integral $\int_0^4 \frac{x}{x^2+1}\, dx$.*

Practice problems: Text §5.5, p. 364: 35, 36, 37, 40

Objective 3: State the integration by parts rule for (in)definite integrals. [Recall]

Reading: Text §7.1, pp. 456 – 457

Objective 4: Apply the integration by parts rule to transform a given (in)definite integral into a more simple (in)definite integral. [Procedural]

Example problem: *Simplify and then evaluate the integral $\int_1^e x^2 \cdot \ln(x)\,dx$.*

Practice problems: Text §7.1, p. 458: 29, 32, 33