Formula Sheet

You may refer to these formulae if necessary.

Summation formulae:

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}, \qquad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}, \qquad \sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}.$$

Trigonometric formulae:

$$\cos^2 x = \frac{1+\cos(2x)}{2}, \qquad \sin^2 x = \frac{1-\cos(2x)}{2}, \qquad \sin(2x) = 2\sin x \cos x.$$

Simpson's rule:

$$S_n = \frac{\Delta x}{3}\Big(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + ... + 4f(x_{n-1}) + f(x_n)\Big).$$

$$E_s = \frac{K(b-a)(\Delta x)^4}{180}, \quad |f^{(4)}(x)| \leq K \text{ on } [a,b].$$

Indefinite Integrals:

$$\int \sec x \, dx = \ln\big|\sec x + \tan x\big| + C.$$

Probability:

- If X is a discrete random variable taking values $x_1, x_2, \cdots, x_m$ with probabilities $p_1, p_2, \cdots, p_m$ respectively, $p_1 + p_2 + \cdots + p_m = 1$, then

$$\mathbb{E}(X) = \sum_{k=1}^{m} x_k p_k, \qquad \mathrm{Var}(X) = \sum_{k=1}^{m} \left[x_k - \mathbb{E}(X)\right]^2 p_k.$$

- If X is a continuous random variable with probability density function $f(x)$, then

$$\mathbb{E}[X] = \int_{-\infty}^{\infty} x f(x)\, dx, \qquad \mathrm{Var}[X] = \int_{-\infty}^{\infty} (x - \mathbb{E}[X])^2 f(x)\, dx.$$

Approximation using Taylor polynomials:

Let n be a fixed positive integer. Suppose there exists a number M such that $|f^{(n+1)}(c)| \leq M$ for all c between a and x inclusive. The remainder in the n^{th}-order Taylor polynomial for f centered at a satisfies

$$|R_n(x)| = |f(x) - p_n(x)| \leq M \frac{|x-a|^{n+1}}{(n+1)!}.$$