Question 1

a. (8 pts) Find constants A, B, C, and D so that

$$\frac{x^3 - 8x + 4}{x(x-1)(x-2)} = A + \frac{B}{x} + \frac{C}{x-1} + \frac{D}{x-2}.$$

b. (8 pts) Compute the indefinite integral

$$\int \sec^3 x \tan^3 x \, dx.$$

c. (8 pts) Suppose Simpson's Rule with $n = 20$ was used to estimate

$$\int_0^\pi x^4 + \sin(2x) \, dx.$$

Find, with justification, a reasonable upper bound for the absolute error of this approximation.

d. (8 pts) Let Z be a continuous random variable with a standard normal distribution. Recall that the probability density function of Z is

$$f(x) = \frac{1}{\sqrt{2\pi}}\, e^{-\frac{1}{2}x^2}.$$

Suppose we define a new function $H(x)$ in terms of the probability

$$H(x) = \Pr(-x^2 < Z < x^2).$$

Find a formula for $H'(x)$. Your final answer must not contain any derivative or integral signs.

e. (8 pts) Consider the improper integral

$$\int_{-1}^{8} x^{-5/3}\, dx.$$

Determine whether this integral converges or diverges. If it converges, then calculate its value.

f. (10 pts) Calculate the following integral using a suitable trigonometric substitution:

$$\int \frac{dx}{x^2\sqrt{x^2-1}}\,dx.$$

Simplify your final answer so that it doesn't contain any trigonometric functions.

Question 2

(15 pts) Compute the definite integral

$$\int_0^{\pi/2} \left(\sin^3 x\right) e^{\cos x}\, dx.$$

Question 3

A certain continuous random variable X is known to have a cumulative distribution function of the form

$$F(x) = \begin{cases} a, & \text{if } x < 0; \\ \frac{1}{2}x^2 + kx, & \text{if } 0 \leq x \leq 1; \\ b, & \text{if } x > 1, \end{cases}$$

where a, b, and k are constants.

a. (6 pts) Determine the exact values of a, b, and k using the fact that $F(x)$ is the CDF of the continuous random variable X.

b. (6 pts) What is the expected value of X?

c. (8 pts) What is the the standard deviation of X?

Question 4

(15 pts) Find the solution of the differential equation

$$\frac{dy}{dt} = e^{y-\ln t}$$

that satisfies the initial condition $y(1) = -1$.