**The University of British Columbia**

Midterm 2 - March 16, 2012

**Mathematics 105, 2011W T2**

**Sections 201, 202, 210**

Closed book examination Time: 50 minutes

**Last Name** ________________ **First** ____________ **SID** ________________

**Instructor names: Edward Kroc, Athena Nguyen, Robert Klinzmann**

**Special Instructions:**

1. A separate formula sheet will be provided. No books, notes, or calculators are allowed. Unless it is otherwise specified, answers may be left in "calculator-ready" form. Simplification of the final answer is worth at most one point.
2. Show all your work. A correct answer without accompanying work will get no credit.
3. If you need more space than the space provided, use the back of the previous page.

**Rules governing examinations**

- Each candidate must be prepared to produce, upon request, a UBCcard for identification.
- Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.
- Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
  (a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners.
  (b) Speaking or communicating with other candidates.
  (c) Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
- Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

| Q | Points | Max |
|---|---|---|
| 1 | | 60 |
| 2 | | 20 |
| 3 | | 20 |
| 4 (extra credit) | | 5 |
| Total | | 100 |

1. (a) The area function of a curve $y = f(t)$ between 0 and $x$ is given by

$$A(x) = 1 - e^{-x^2}.$$

Find the slope of the tangent to the curve $y = f(t)$ at the point $t = 1$.

(10 points)

(b) Use Simpson's rule to approximate

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x \, dx$$

with $n = 4$ subintervals. Find a bound on the error. **No need to simplify your answers!**

(5 + 5 = 10 points)

(c) Find the definite integral

$$\int_0^{\pi} \sec x \tan x \, dx.$$

(10 points)

(d) A game consists of drawing balls from two urns containing two balls each. The balls in the first urn are marked 1 and 2, the balls in the second urn are marked 3 and 4. A player picks two balls simultaneously, one from each urn. If the sum of the numbers obtained is even, he wins an amount equal to the sum. If the sum is odd, he loses that amount. Find the expected value of the player's gain.

(10 points)

(e) Obtain the partial fraction decomposition of the function

$$\frac{x+3}{x^2+3x+2}.$$

(10 points)

(f) Solve the initial value problem

$$y' = y^2 - e^{3t}y^2, \qquad y(0) = 1.$$

(10 points)

2. Evaluate the definite integral:

$$\int_0^3 \frac{\arctan(x/3)}{(9+x^2)^{\frac{3}{2}}}\,dx.$$

(20 points)

3. The proportion of new stores on a boulevard that make a profit during their first year is given by a continuous random variable $X$ whose probability density function is given by

$$f(x) = \begin{cases} kx(1-x)^2 & \text{if } 0 \le x \le 1, \\ 0 & \text{otherwise.} \end{cases}$$

(10 + 10 = 20 points)

(a) Find the value of $k$.

(b) What is the probability that no more than half the stores make a profit in year one?

4. (Extra credit) A certain item of news is being broadcast to a potential audience of 200,000 people. At any given time, 7% of the population who have not heard the news switches on the TV and is informed of it. Initially, none of the audience members were aware of the news. Write down an initial value problem that models the spread of news in the population. **Do not solve this problem!**

(5 points)

## Formula Sheet

You may refer to these formulae if necessary.

**Trigonometric formulae:**

$$\cos^2 x = \frac{1+\cos(2x)}{2}.$$
$$\sin^2 x = \frac{1-\cos(2x)}{2}.$$

**Simpson's rule:**

$$S_n = \frac{\Delta x}{3}\Big(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + ... + 4f(x_{n-1}) + f(x_n)\Big).$$
$$E_s = \frac{K(b-a)(\Delta x)^4}{180}, \quad |f^{(4)}(x)| < K \text{ on } [a,b].$$

**Indefinite Integrals:**

$$\int \sec x\, dx = \ln\big|\sec x + \tan x\big| + C.$$

**Probability:**

$$\mathbb{E}[X] = \int_{-\infty}^{\infty} x f(x)\, dx.$$
$$\mathrm{Var}[X] = \int_{-\infty}^{\infty} (x - \mathbb{E}[X])^2 f(x)\, dx.$$