The University of British Columbia

Midterm 2 - March 16, 2012

Mathematics 105, 2011W T2

Section 203

Closed book examination Time: 50 minutes

Last Name _____ **First** _____ **SID** _____

Instructor name: Keqin Liu

Special Instructions:

1. A separate formula sheet will be provided. No books, notes, or calculators are allowed. Unless it is otherwise specified, answers may be left in "calculator-ready" form. Simplification of the final answer is worth at most one point.
2. Show all your work. A correct answer without accompanying work will get no credit.
3. If you need more space than the space provided, use the back of the previous page.

Rules governing examinations

- Each candidate must be prepared to produce, upon request, a UBCcard for identification.
- Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.
- Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
  - (a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners.
  - (b) Speaking or communicating with other candidates.
  - (c) Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
- Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

Q	Points	Max
1		60
2		20
3		20
4 (extra credit)		5
Total		100

1. (a) Find the derivative of the function

$$f(x) = \int_x^{x^2} \cos(t^2)\, dt$$

at the point $x = 0$.

(10 points)

(b) Use Simpson's rule to approximate

$$\int_1^2 \frac{dx}{x}$$

with $n = 4$ subintervals. Find a bound on the error. **No need to simplify your answers!**

(5 + 5 = 10 points)

(c) Find the definite integral

$$\int_0^{\pi} \sec^2 x \, dx.$$

(10 points)

(d) A student randomly guesses at each answer in a true/false quiz consisting of 3 questions. Let X be the random variable representing the number of correct answers. Find the probability density function of X.

(10 points)

(e) Find the indefinite integral

$$\int \sin^3(x)\cos^{10}(x)\,dx.$$

(10 points)

(f) Solve the initial value problem

$$e^{-t}y' = \frac{t}{y}, \qquad y(0) = -5.$$

(10 points)

2. Evaluate the definite integral:

$$\int_0^{\frac{\ln(\sqrt{3})}{2}} \frac{e^{2t}}{(1+e^{4t})^{\frac{3}{2}}}\, dt.$$

(20 points)

3. The time to failure of a transistor (in years) is a continuous random variable whose cumulative distribution function is given by

$$F(x) = \begin{cases} 1 - e^{-mx} & \text{if } x \geq 0, \\ 0 & \text{otherwise,} \end{cases}$$

where m is an unknown constant.

$(3 + 7 + 10 = 20$ points$)$

(a) Find the probability density function of X.

(b) If the expected time to failure of a transistor is 10, find m.

(c) What is the probability that a transistor will last for at least 15 years?

4. (Extra credit) The monthly average price of silver has been growing at a rate proportional to the square root of the price since November 2011. The average price in November 2011 of one gram of silver was \$16 and the average price of the same in February 2012 was \$25. Write down the initial value problem of the monthly value of silver as a function of time. **Do not solve this problem!**

(5 points)

Formula Sheet

You may refer to these formulae if necessary.

Trigonometric formulae:

$$\cos^2 x = \frac{1+\cos(2x)}{2}.$$
$$\sin^2 x = \frac{1-\cos(2x)}{2}.$$

Simpson's rule:

$$S_n = \frac{\Delta x}{3}\Big(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + ... + 4f(x_{n-1}) + f(x_n)\Big).$$
$$E_s = \frac{K(b-a)(\Delta x)^4}{180}, \quad |f^{(4)}(x)| < K \text{ on } [a,b].$$

Indefinite Integrals:

$$\int \sec x \, dx = \ln\big|\sec x + \tan x\big| + C.$$

Probability:

$$\mathbb{E}[X] = \int_{-\infty}^{\infty} x f(x)\, dx.$$
$$\mathrm{Var}[X] = \int_{-\infty}^{\infty} (x - \mathbb{E}[X])^2 f(x)\, dx.$$