The University of British Columbia

Midterm 2 - March 16, 2012

Mathematics 105, 2011W T2

Sections 204, 205, 206, 211

Closed book examination Time: 50 minutes

Last Name ________________ **First** ____________ **SID** ________________

Instructor names: Malabika Pramanik, Paul Pollack, Keqin Liu, Erick Wong

Special Instructions:

1. A separate formula sheet will be provided. No books, notes, or calculators are allowed. Unless it is otherwise specified, answers may be left in "calculator-ready" form. Simplification of the final answer is worth at most one point.
2. Show all your work. A correct answer without accompanying work will get no credit.
3. If you need more space than the space provided, use the back of the previous page.

Rules governing examinations

- Each candidate must be prepared to produce, upon request, a UBCcard for identification.
- Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.
- Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
  (a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners.
  (b) Speaking or communicating with other candidates.
  (c) Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
- Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

Q	Points	Max
1		60
2		20
3		20
4 (extra credit)		5
Total		100

1. (a) Find the derivative of the function

$$f(x) = x^2 \int_3^x t \sin\left(\frac{\pi t}{6}\right)\, dt$$

at the point $x = 3$.

(10 points)

(b) Use Simpson's rule to approximate

$$\int_1^2 \ln x \, dx$$

with $n = 4$ subintervals. Find a bound on the error. **No need to simplify your answers!**

(5 + 5 = 10 points)

(c) Find the definite integral

$$\int_{-2}^{1} \frac{2}{(x+1)^4}\, dx.$$

(10 points)

(d) We have a pair of fair coins. The faces of the first coin are marked with the numbers 0 and 1, the faces of the second coin are marked with the numbers +2 and -2. We toss both coins simultaneously. Let X denote the random variable given by the product of values that appear face up. Find the probability density function of X.

(10 points)

(e) Obtain the partial fraction decomposition of the function

$$\frac{x-7}{x^2-x-12}.$$

(10 points)

(f) Solve the initial value problem

$$3y' + y^4 \cos t = 0, \quad y\left(\frac{\pi}{2}\right) = \frac{1}{2}.$$

(10 points)

2. Evaluate the definite integral:

$$\int_0^{\frac{\pi}{2}} e^{\sin x} (\sin x + 1) \cos x \, dx.$$

(20 points)

3. Let X be a continuous random variable that measures the lifetime (in years) of a light bulb. It can be shown that the probability density function of X is given by

$$f(x) = \begin{cases} \frac{1}{2}e^{-\frac{x}{2}} & \text{if } x \geq 0, \\ 0 & \text{otherwise.} \end{cases}$$

(10+10 = 20 points)

(a) Compute the cumulative distribution function of X.

(b) Find the expected value of X.

4. (Extra credit) A bank account earns 10% annual interest with continuous compounding. This means that at any given time, the account earns interest at a rate that is 10% of the account balance at that time. In addition, money is deposited into the account at the rate of $1200 per year, spread evenly throughout the year. If the initial balance is $0, write down an initial value problem that models the account balance as a function of time. **Do not solve this problem!**

(5 points)

Formula Sheet

You may refer to these formulae if necessary.

Trigonometric formulae:

$$\cos^2 x = \frac{1+\cos(2x)}{2}.$$
$$\sin^2 x = \frac{1-\cos(2x)}{2}.$$

Simpson's rule:

$$S_n = \frac{\Delta x}{3}\Big(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + ... + 4f(x_{n-1}) + f(x_n)\Big).$$
$$E_s = \frac{K(b-a)(\Delta x)^4}{180}, \quad |f^{(4)}(x)| < K \text{ on } [a,b].$$

Indefinite Integrals:

$$\int \sec x\, dx = \ln\big|\sec x + \tan x\big| + C.$$

Probability:

$$\mathbb{E}[X] = \int_{-\infty}^{\infty} x f(x)\, dx.$$
$$\mathrm{Var}[X] = \int_{-\infty}^{\infty} (x - \mathbb{E}[X])^2 f(x)\, dx.$$