The University of British Columbia

Midterm 1 - February 3, 2012

Mathematics 105, 2011W T2

Sections 201, 202, 210

Closed book examination   Time: 50 minutes

Last Name _____ **First** _____ **SID** _____

Instructor names: Edward Kroc, Athena Nguyen, Robert Klinzmann

Special Instructions:

1. A separate formula sheet will be provided. No books, notes, or calculators are allowed. Unless it is otherwise specified, answers may be left in "calculator-ready" form. Simplification of the final answer is worth at most one point.

2. Show all your work. A correct answer without accompanying work will get no credit.

3. If you need more space than the space provided, use the back of the previous page. Where boxes are provided for answers, put your final answers in them.

Rules governing examinations

Q	Points	Max
1		50
2		20
3		20
4		10
5 (extra credit)		10
Total		100

1. (a) Let
$$f(x, y) = \sqrt{x^2 + 2y^2}.$$

Use $f_x(1, 2)$ and $f_y(1, 2)$ to estimate $f(1.3, 1.7)$.

(8 points)

(b) Let $\mathbf{v} = \langle a, b \rangle$ and $\mathbf{w} = \langle -b, a \rangle$, where a, b are arbitrary real numbers, at least one of which is nonzero. Are the two vectors $\mathbf{v}$ and $\mathbf{w}$ always perpendicular, irrespective of the choices of a and b?

(8 points)

(c) Find a unit vector parallel to $\langle 3, \sqrt{8}, -\sqrt{8} \rangle$.

(8 points)

(d) Compute the left Riemann sum with three equal subintervals for $f(x) = 15/x$ in the interval $[1, 7]$.

(8 points)

(e) Is there a function $f(x, y)$ such that $\nabla f(x, y) = \langle 2xy, x^2 + xy \rangle$? If not, explain why no such function exists; otherwise find $f(x, y)$. State clearly any result that you use.

(8 points)

(f) Let R be the semicircular region $\{x^2 + y^2 \leq 4, \ x \leq 0\}$. Find the maximum and minimum values of the function

$$f(x, y) = x^2 + y^2 + 2x$$

on the *boundary of the region R*.

(10 points)

2. Find *all* critical points of the function

$$f(x, y) = \frac{x^3}{3} - \frac{y^3}{3} + 3xy$$

Classify each point as a local maximum, local minimum, or saddle point.

$$(10 + 10 = 20 \text{ points})$$

3. A cookie company produces sugar-free cookies, where each cookie contains s milligrams of sucralose and t milligrams of acesulfame. Market studies show that consumer satisfaction is best if the sweetness level of the cookie is maintained according to the equation

$$\sqrt{s} + \sqrt{t} = 5.$$

If the price of sucralose is \$20 per gram and the price of acesulfame is only \$5 per gram, use Lagrange multipliers to find how the company should choose s and t so as to minimize costs. (Remember, there are 1000 milligrams in one gram).

Clearly state the objective function and the constraint. There is no need to justify that the solution you obtained is the absolute max or min. **A solution that does not use the method of Lagrange multipliers will receive no credit, even if it is correct.**
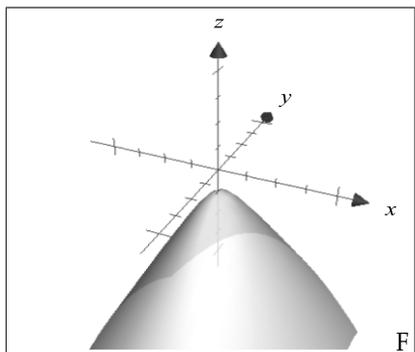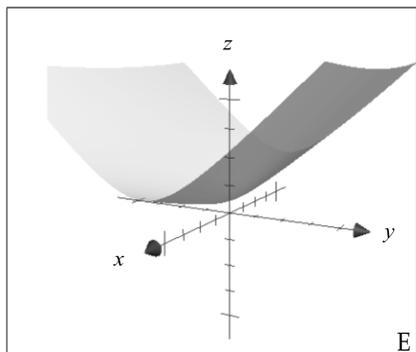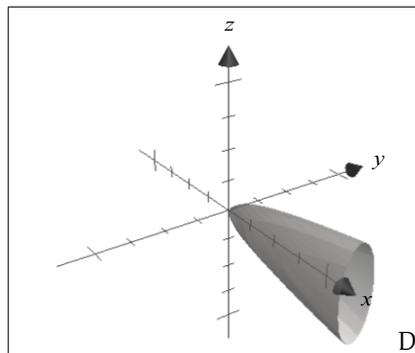
(20 points)

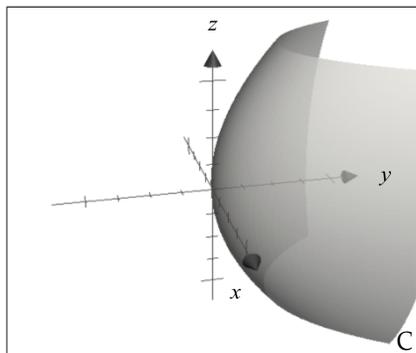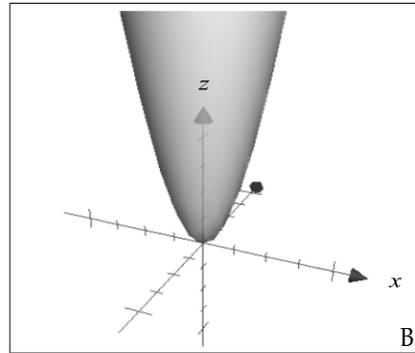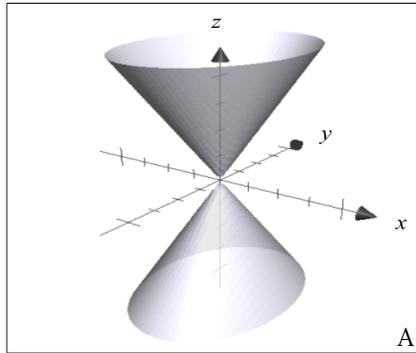4. Consider the surface S given by

$$y - \frac{x^2}{9} = \frac{z^2}{25}.$$

(a) Sketch the traces of S in the $y = 4$ and $z = 0$ planes.

$(3 + 3 = 6 \text{ points})$

(b) Based on the traces you sketched above, which of the following renderings represents the graph of the surface?

(4 points)

A

B

C

D

E

F

1

5. (Extra credit) Transform the limit of the following Riemann sum to a definite integral:

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{2}{n} \cos\left(1 + \frac{4k}{n}\right).$$

(10 points)

Formula Sheet

You may refer to these formulae if necessary.

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}.$$