

The University of British Columbia

Midterm 1 - February 3, 2012

Mathematics 105, 2011W T2

Sections 204, 205, 206, 211

Closed book examination

Time: 50 minutes

Last Name _____ First _____ SID _____

Instructor names: Malabika Pramanik, Paul Pollack, Keqin Liu, Erick Wong

Special Instructions:

1. A separate formula sheet will be provided. No books, notes, or calculators are allowed. Unless it is otherwise specified, answers may be left in “calculator-ready” form. Simplification of the final answer is worth at most one point.
2. Show all your work. A correct answer without accompanying work will get no credit.
3. If you need more space than the space provided, use the back of the previous page. Where boxes are provided for answers, put your final answers in them.

Rules governing examinations

- Each candidate must be prepared to produce, upon request, a UBCCard for identification.
- Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.
- Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
 - (a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners.
 - (b) Speaking or communicating with other candidates.
 - (c) Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
- Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

Q	Points	Max
1		50
2		20
3		20
4		10
5 (extra credit)		10
Total		100

1. (a) Let

$$f(x, y) = x \sin(y).$$

If the value of (x, y) changes from $(0, \frac{\pi}{2})$ to $(0.1, \frac{\pi}{2} - 0.2)$, estimate the corresponding change in the value of f .

(8 points)

(b) Let $\mathbf{v} = \langle \frac{4}{3}, -\frac{1}{2} \rangle$ and $\mathbf{w} = \langle 16, -6 \rangle$. Are the two vectors \mathbf{v} and \mathbf{w} parallel, perpendicular or neither? Justify your answer.

(8 points)

- (c) A plane \mathcal{P} is parallel to the plane $2x - y + z = 0$ and passes through the point $P_0(1, -1, 3)$. Find the equation of the plane \mathcal{P} .

(8 points)

- (d) Compute the right Riemann sum with four equal subintervals for $f(x) = 16x^2$ in the interval $[0, 1]$.

(8 points)

(e) Given functions

$$F(x, y) = x + e^y, \quad \text{and} \quad G(x, y) = y + e^x,$$

does there exist a function $f(x, y)$ such that $\nabla f(x, y) = \langle F, G \rangle$? Justify your answer in detail, citing any result that you use.

(8 points)

- (f) Let R be the semicircular region $\{x^2 + y^2 \leq 4, y \geq 0\}$. Find the maximum and minimum values of the function

$$f(x, y) = x^2 + y^2 - 3y$$

on the *boundary of the region* R .

(10 points)

2. Find *all* critical points of the function

$$f(x, y) = 3x^2 + 6xy - 2y^3$$

Classify each point as a local maximum, local minimum, or saddle point.

(10 + 10 = 20 points)

3. A candy company produces boxes of bubblegum and gummy bears. It costs the company \$1 to produce a box of either type of candy. On the other hand, bubblegum sells for \$3 per box and gummy bears for \$5 per box. Due to limitations of sugar supply, the production scheme has to satisfy the production possibilities curve

$$\sqrt{x} + 2\sqrt{y} = 300,$$

where x and y denote the number of boxes of bubblegum and gummy bears that the company produces weekly. Assuming that the company manages to sell every unit produced, use the method of Lagrange multipliers to answer the following question: how many boxes of each type of candy should the company aim to produce if it is to maximize profit?

Clearly state the objective function and the constraint. There is no need to justify that the solution you obtained is the absolute max or min. **A solution that does not use the method of Lagrange multipliers will receive no credit, even if it is correct.**

(20 points)

4. Consider the surface S given by

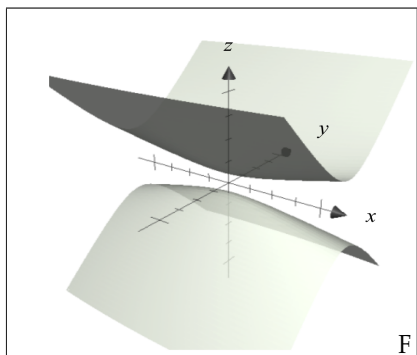
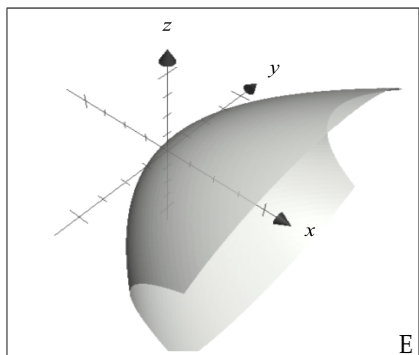
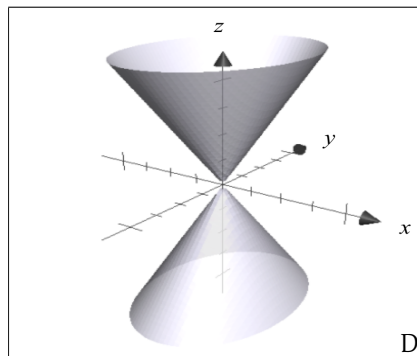
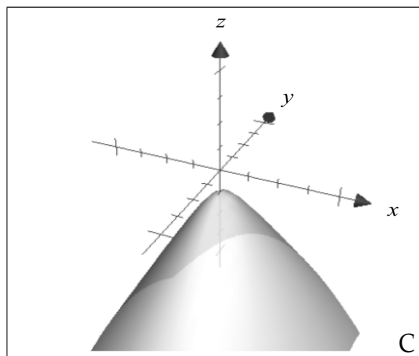
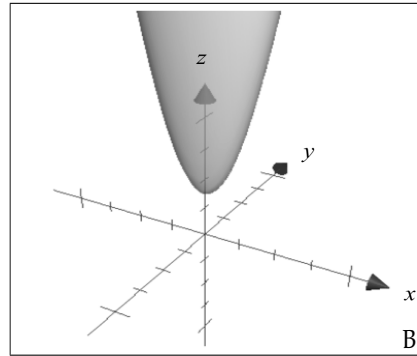
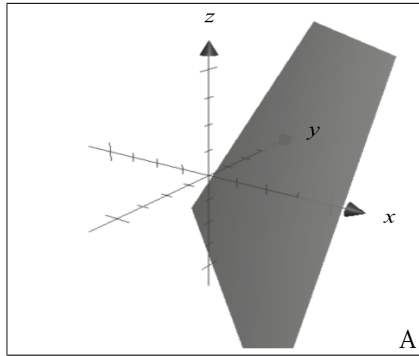
$$z = 3 + x^2 + 4y^2.$$

(a) Sketch the traces of S in the $z = 4$ and $x = 0$ planes, labeling the axes carefully.

(3 + 3 = 6 points)

(b) Based on the traces you sketched above, which of the following renderings represents the graph of the surface?

(4 points)



5. (Extra credit) Transform the limit of the following Riemann sum to a definite integral:

$$\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{2}{n} \left(1 + \frac{2k}{n} \right)^5 .$$

(10 points)

Formula Sheet

You may refer to these formulae if necessary.

$$\sum_{k=1}^n k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^n k^3 = \frac{n^2(n+1)^2}{4}.$$