Math 105 Week 7
February 21st, 2011

1. Lesson plan

This week we will begin to study functions of several variables. We will see that concepts like graphs, continuity, (and later derivatives,) can be defined for such functions. However, these concepts are more complicated than their single variable analogs.

We start in section 12.1 by defining equations of planes in $\mathbb{R}^3$. These objects are defined by a single linear equation in three variables. We will also introduce the notion of a *vector* in $\mathbb{R}^3$, which will provide useful geometric intuition behind many of concepts we will learn in the following week.

We will also consider *Quadric surfaces*. These are surfaces in $\mathbb{R}^3$ that are a two-dimensional analog of many familiar curves in $\mathbb{R}^2$ (circles, ellipses, parabolas, and hyperbolas).

Next, in section 12.2, we will cover drawing graphs of functions of two variables, and introduce the notion of *level curves*. We will use level curves to represent a surface in $\mathbb{R}^3$ as a union of many different level curves. Since drawing a curve is easier than drawing a surface, this enables us to reduce the problem of drawing the graph of a function of two variables into drawing a system of curves.

Finally, in section 12.3, we will introduce the notion of continuity for a function of two variables. This definition is similar to that for single variable calculus (see section 2.6 for a reminder), but we'll see that the continuity in two variables is much more complicated than in the single variable situation. Our main tool for checking discontinuity will be the *two-path test*, a two-dimensional analog of comparing left-hand and right-hand limits for function of one variable (see page 59). When the limit does exist, we will evaluate these limits by a combination of the following techniques: (i) substitution, (ii) factoring, and (iii) derivatives.

2. Learning objectives

By the end of the week and after going through the practice problems, you should be able to:

1. write down the equation of a plane in $\mathbb{R}^3$, given a point on the plane and a vector normal to the plane. Also, you should be able to recognize when two such equations produce planes that are parallel.
2. understand the notational and conceptual distinction between a vector and a point.
3. match the picture and equation for ellipsoids, paraboloids, hyperboloids, cylinders, and cones (see page 785.)
4. find the domain and range of a given function $f(x, y)$.
5. draw level curves for a function of two variables, and relate the level curves to the graph.
6. know the definition of continuity for a function of two variables
7. use the two-path test to check for non-existence of limits.