Math 105 Practice Exam 3

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1. (a) Evaluate $\displaystyle\lim_{(x,y)\to(1,-1)} \frac{\sin(x^2+y)}{x^2+y}$ or show that it doesn't exist.

   (b) Consider the area function $A(x) = \int_1^x f(t)dt$, with $A(2) = 6$ and $A(3) = 5$. Compute $\displaystyle\int_3^2 f(t)dt$.

   (c) A self-employed software engineer estimates that her annual income over the next 10 years will steadily increase according to the formula $70,000e^{0.1t}$, where t is the time in years. She decides to save 10% of her income in an account paying 6% annual interest, compounded continuously. Treating the savings as a continuous income stream over a 10-year period, find the present value.

   (d) Draw the level curves of the graph of $f(x,y) = 2x^2 + y^2$ at the heights $0, 1, 2$.

   (e) Evaluate $\displaystyle\int_0^1 \frac{\cos(\sqrt{x})}{\sqrt{x}}dx$.

   (f) Let $f(x,y) = \dfrac{x+y}{x-y}$. Use linear approximation to estimate $f(2.95, 2.05)$.

2. Evaluate $\displaystyle\int \frac{x+2}{x(x^2-1)}dx$.

3. Find the area of the region in the first quadrant bounded by $y = \frac{1}{x}$, $y = 4x$, and $y = \frac{1}{2}x$.

4. Find k such that $f(x) = \frac{k}{(x+1)^3}$ is a probability density function on the interval $[0, \infty)$, for some random variable X. Then compute the probability that $1 \le X \le 4$.

5. Mothballs tend to evaporate at a rate proportional to their surface area. If V is the volume of a mothball, then its surface area is roughly $V^{2/3}$. Suppose that the mothball's volume $V(t)$ (as a function of times t in weeks) decreases at a rate that is twice its surface area, and that it initially has a volume of 27 cubic centimeters.
   Construct and solve an initial value problem for the volume $V(t)$. Then determine if and when the mothball vanishes.

6. Consider the surface $z = f(x,y) = 1 + \frac{1}{\sqrt{xy}}$. At the point on the surface above the point $(x,y) = (4,1)$, what is the direction of steepest descent? Describe this direction with a unit vector in the xy-plane.

7. By employing x semi-skilled workers and y skilled workers, a factory can assemble $\sqrt{4xy + y^2}$ custom-built computers per hour. The factory pays each semi-skilled worker \$8 per hour, and each skilled worker \$20 per hour. Determine the maximum number of computers the factory can assemble in an hour for a total labour cost of \$720.

8. Find and classify the critical points of $f(x,y) = 7x^2 - 5xy + y^2 + x - y + 6$.

9. Given the supply and demand curves
   $$p = D(q) = 8 - q, \qquad p = S(q) = \sqrt{q+1} + 3,$$
   find the equilibrium point and the consumer/producer surplus.