

## Math 105 Practice Final Exam 1

1(a). Evaluate the limit  $\lim_{(x,y) \rightarrow (0,0)} \frac{xy^{1/2}}{x^2-y}$  if the limit exists.

(b). Suppose  $F(x) = \int_1^{e^{x^2}} f(t)dt$  and  $f(e) = 4$ . Find  $F'(1)$ .

(c). Which of the following equations describes an elliptic paraboloid. Find the equations of and sketch the level curves corresponding to height 0, 1, -1 if there is any. (A).  $z = \frac{x^2}{4} + \frac{y^2}{9}$ . (B).  $z = \frac{x^2}{4} - \frac{y^2}{9}$ . (C).  $z^2 + 1 = \frac{x^2}{4} + \frac{y^2}{9}$ .

(d). Evaluate  $\int_0^\infty xe^{-x^2} dx$ , if it is convergent.

(e). Use linear approximation to find the distance between (2.98, 4.01) and the origin point.

2(a). Evaluate  $\int_0^{\pi/2} e^x \sin x dx$ .

(b). Use trapezoid rule with  $n = 2$  to approximate the above definite integral.

3. Find the area of the region bounded by  $y = x^2$ ,  $y = x^2 - 4x + 4$ ,  $x = 0$  and  $x = 3$ .

4. Suppose a kind of deer in a region has no natural enemy and its number grows exponentially at a rate of 12% per year. To control the number of the deer, it is allowed to hunt  $A$  deers. Suppose at the beginning there are 50,000 deers. Let  $y(t)$  denote the number of deers at time  $t$  year. (a). write down an initial value problem satisfied by  $y(t)$ . (b). Specify  $A$  so that the number of deers remain constant. (c). If actually 7200 deers were hunted due to poaching, how long does it take for the deer to extinct?

5. The HigherPrices corporation (HP) has a local monopoly on printers and the proprietary ink cartridges for them. The selling price  $f(x, y)$ , in dollars, of a printer is connected to the number  $x$  of printers and the number of  $y$  of cartridges sold per week by the equation  $f(x, y) = 300 - x$ . Note there is no dependence on  $y$  shoppers do not think about the cost of cartridges when buying a printer. The cost to HP of a printer is \$200.

The selling price  $g(x, y)$ , in dollars, of a cartridge is connected to the number  $x$  of printers and the number  $y$  of cartridges sold per week by the equation  $g(x, y) = 61 - \frac{3y}{x}$ . The cost to HP of a cartridge is \$1.

How many printers and cartridges should HP sell to maximize its weekly profit? Do not justify that your answer produces maximal profit.

6. The amount of space required by a particular firm is  $f(x, y) = 1000\sqrt{6x^2 + y^2}$ , where  $x$  and  $y$  are, respectively, the number of units of labor and capital utilized. Suppose that labor costs \$480 per unit and capital costs \$40 per unit and that the firm has \$5000 to spend. Determine the amounts of labor and capital that should be utilized in order to minimize the amount of space required.

7. Suppose  $D(q) = 0.05(q - 10)^2$ ,  $S(q) = 0.15q + 2$ ,  $0 \leq q \leq 10$  are demand and supply functions respectively. Find the equilibrium point and consumer surplus.

8. Suppose  $f(x, y) = \frac{e^x}{1+e^y}$ . At point  $(0, 0)$ , find the maximal increasing rate and the corresponding direction.

9. Find constant  $K$  such that  $f(x) = K|\cos(x)|$  is a probability density function on  $0 \leq x \leq \pi$  for some random variable.