

## Math 105 Practice Midterm2, Spring 2011

### 1. Short answer questions

- (1) Evaluate  $\int_1^\infty \frac{\arctan x}{x^2+1} dx$ .
- (2). Solve the initial value problem  $\frac{dx}{dt} = \frac{x^2+5}{x}$ ,  $x(0) = 1$ .
- (3). Evaluate  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2y+xy^2}{y^3-x^3}$ .
- (4). Let  $z = \sqrt{x - y^2 - 1}$ . Sketch the level curve through the point  $(6, 1)$ . Calculate the slope of the tangent line to this level curve at  $(6, 1)$ .

### 2. Find and classify the critical points of $f(x, y) = x^4 + 2y^2 - 4xy$ .

3. A person purchased a home at the price \$300,000, paid a down payment equal to 20% of the purchase price, and financed the remaining balance with a 25 year term mortgage. Assume that the person makes payments continuously at a constant annual rate  $A$  and that interest is compounded continuously at the rate of 5%.

(a) Write down the differential equation that is satisfied by the amount  $y(t)$  of money owed on the mortgage at time  $t$ .

(b) Determine  $A$ , the rate of annual payments, that is required to pay off the loan in 25 years.

(c) Determine the total interest paid during the 25 year term mortgage.

### 4. Consider the hill given by the function $z = f(x, y) = \sqrt{4 - x^2 - \frac{y^2}{4}}$ .

(a) Compute  $f_x$  and  $f_y$ .

(b) Find the unit vector that gives the direction of steepest ascent at the point  $P(1, 2)$ . Also find a unit vector that gives the direction of no change at that point. Sketch these two vectors and the level curve through  $P$ .

(c) Suppose you're walking over the hill along the path that is right above the path  $(x(t), y(t)) = (t, t^2 + 1)$  in the  $xy$ -plane. As you pass the point  $(1, 2, f(1, 2))$ , at what rate is your height changing?

### 5. Find the area of the shaded region bounded by the curves $f(x) = x^3 - 4x$ and $g(x) = -x^2 + 2x$ for $x \geq 0$ .