

Math 105

Practice Midterm 1 for Midterm 2

This practice midterm may be harder and/or longer than the real midterm.

Not all question will be worth the same number of points.

1. Evaluate $\int_0^{\infty} \frac{e^{-\sqrt{x}}}{\sqrt{x}} dx$, or show that it doesn't exist.
 2. Solve the initial value problem $y' = \frac{1}{\sqrt{xy}}$, $y(1) = 4$.
 3. Find an equation for the plane that is parallel to $x - 2y + 6z = 1$ and contains the point $(4, 0, 2)$.
 4. Sketch the level curves of $z = y^2 - \frac{1}{4}x^2$ at the heights $z = -1, 0, 1$.
 5. Evaluate the limit $\lim_{(x,y) \rightarrow (0,0)} \frac{5x - 2y^2}{x + 2y^2}$, or show that it doesn't exist.
 6. Consider the hill given by the function $z = f(x, y) = \sqrt{1 - x^2 - 4y^2}$.
 - (a) Compute f_x and f_y .
 - (b) Find the unit vector that gives the direction of steepest ascent at the point $(\frac{1}{2}, \frac{1}{4}, f(\frac{1}{2}, \frac{1}{4}))$ on the hill. Also find a unit vector that gives the direction of no change at that point.
 - (c) Suppose you're walking over the hill along the path that is right above the path $(x(t), y(t)) = (t, t^2)$ in the xy -plane. As you pass the point $(\frac{1}{2}, \frac{1}{4}, f(\frac{1}{2}, \frac{1}{4}))$, at what rate is your height changing?
 7. Find the critical points of $f(x, y) = \frac{1}{2}x^2 + 4xy + y^3 + 8y^2 + 3x + 2$, and classify each one as a maximum, minimum or saddle point.
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