4. $\lim_{n\to\infty} \frac{(-1)^n n^2}{\pi n(n-\pi)} = \lim_{n\to\infty} \frac{(-1)^n}{1-(\pi/n)}$ does not exist.

The sequence diverges (oscillates).

18. $\sum_{n=1}^{\infty} \frac{(-1)^n}{2^n - n}$ converges absolutely by comparison with the convergent geometric series $\sum_{n=1}^{\infty} \frac{1}{2^n}$, because

$$\lim_{n\to\infty} \frac{\left|\frac{(-1)^n}{2^n - n}\right|}{\frac{1}{2^n}} = \lim_{n\to\infty} \frac{1}{1 - \frac{n}{2^n}} = 1.$$

19. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\ln \ln n}$ converges by the alternating series test, but the convergence is only conditional since $\sum_{n=1}^{\infty} \frac{1}{\ln \ln n}$ diverges to infinity by comparison with the divergent harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$. (Note that $\ln \ln n < n$ for all $n \geq 1$.)

10. Since $r^2 = 2\cos 2\theta$ meets $r = 1$ at $\theta = \pm\dfrac{\pi}{6}$ and $\pm\dfrac{5\pi}{6}$, the area inside the lemniscate and outside the circle is

$$4 \times \frac{1}{2}\int_0^{\pi/6}\left[2\cos 2\theta - 1^2\right] d\theta$$

$$= 2\sin 2\theta\Big|_0^{\pi/6} - \frac{\pi}{3} = \sqrt{3} - \frac{\pi}{3} \text{ sq. units.}$$

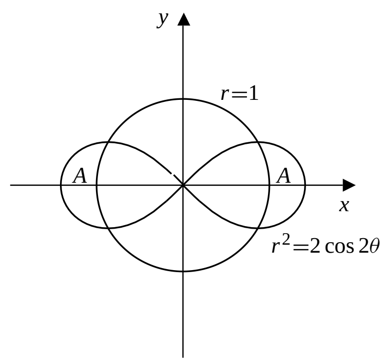

Fig. 6-10

6. $r = \sec\theta\ \tan\theta \Rightarrow r\cos\theta = \dfrac{r\sin\theta}{r\cos\theta}$

$x^2 = y$ a parabola.

28. Let $r_1(\theta) = \theta$ and $r_2(\theta) = \theta + \pi$. Although the equation $r_1(\theta) = r_2(\theta)$ has no solutions, the curves $r = r_1(\theta)$ and $r = r_2(\theta)$ can still intersect if $r_1(\theta_1) = -r_2(\theta_2)$ for two angles θ_1 and θ_2 having the opposite directions in the polar plane. Observe that $\theta_1 = -n\pi$ and $\theta_2 = (n-1)\pi$ are two such angles provided n is any integer. Since

$$r_1(\theta_1) = -n\pi = -r_2((n-1)\pi),$$

the curves intersect at any point of the form $[n\pi, 0]$ or $[n\pi, \pi]$.

22. If $x = f(t) = at - a\sin t$ and $y = g(t) = a - a\cos t$, then the volume of the solid obtained by rotating about the x-axis is

$$\begin{aligned}
V &= \int_{t=0}^{t=2\pi} \pi y^2\, dx = \pi \int_{t=0}^{t=2\pi} [g(t)]^2 f'(t)\, dt \\
&= \pi \int_0^{2\pi} (a - a\cos t)^2 (a - a\cos t)\, dt \\
&= \pi a^3 \int_0^{2\pi} (1 - \cos t)^3\, dt \\
&= \pi a^3 \int_0^{2\pi} (1 - 3\cos t + 3\cos^2 t - \cos^3 t)\, dt \\
&= \pi a^3 \left[2\pi - 0 + \frac{3}{2} \int_0^{2\pi} (1 + \cos 2t)\, dt - 0 \right] \\
&= \pi a^3 \left[2\pi + \frac{3}{2}(2\pi) \right] = 5\pi^2 a^3 \text{ cu. units.}
\end{aligned}$$

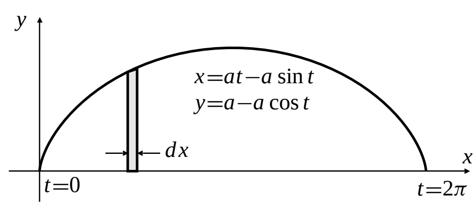

Fig. 4-22