

## Math 121 Assignment 7

Due Friday March 19

### ■ Practice problems:

- Try out as many problems from Chapter 8 as you can, with special attention to the ones marked as challenging problems. As a test of your understanding of the material, work out the problems given in the chapter review. You may skip the ones that require computer aid.

### ■ Problems to turn in:

1. Identify the parametric curves

$$\begin{cases} x = \cosh t \\ y = \sinh^2 t \end{cases} \quad \text{and} \quad \begin{cases} x = \cos t + \sin t, \\ y = \cos t - \sin t. \end{cases}$$

2. For the following two examples, determine the points where the given parametric curves have horizontal and vertical tangents.

$$\begin{cases} x = \frac{4}{1+t^2} \\ y = t^3 - 3t \end{cases} \quad \text{and} \quad \begin{cases} x = t^3 - 3t, \\ y = t^3 - 12t. \end{cases}$$

3. Find the length of the curve  $x = e^t - t$ ,  $y = 4e^{t/2}$  from  $t = 0$  to  $t = 2$ .
4. Sketch the polar graph of the equation  $r = 1 + 2 \cos 2\theta$  and find the area of one of the two smaller loops.
5. Find the area of the region inside the cardioid  $r = 1 + \cos \theta$  and to the left of the line  $x = 1/4$ .
6. Show that a plane that is not parallel to the axis of a circular cylinder intersects the cylinder in an ellipse.
7. At what points do the curves  $r^2 = 2 \sin 2\theta$  and  $r = 2 \cos \theta$  intersect? At what angle do the curves intersect at each of these points?
8. Find the equation of a curve that passes through the point  $(2, 4)$  and has slope  $3y/(x - 1)$  at any point  $(x, y)$  on it.
9. A tractrix is a curve in the first quadrant of the  $(x, y)$  plane, starting from the point  $(L, 0)$ , and having the property that if the tangent line to the curve at  $P$  meets the  $y$ -axis at  $Q$ , then the length of  $PQ$  is the constant  $L$ . (For example, think of a trailer of length  $L$  attached to a tractor which is sitting at the origin. The rear end  $P$  of the trailer was originally lying at  $(L, 0)$ . As the tractor moves away along the  $y$ -axis, the path traced out by  $P$  is a tractrix.) Find the equation of the tractrix.

10. (a) Show that

$$f(x) = \frac{2}{\pi(1+x^2)}$$

is a probability density on  $[0, \infty)$ . Find the expectation of  $X$  for this density.

- (b) A machine generates values of a random variable  $X$  distributed with density  $f(x)$ . You pay a certain fee to play a game where you operate the machine and win  $X$  dollars if the machine shows  $X$ . How much would you be willing to pay to enter the game?