Math 121 Assignment 1

Due Friday January 15

■ Practice problems from the text (do NOT turn in):

- Section 5.1: 7–33, 37–39.
- Section 5.2: 1–19.
- Section 5.3: 5, 7–17.
- Section 5.4: 1–32, 34, 35, 37, 38, 41, 42.

■ Problems to turn in:

1. (a) Write the sum

$$2^2 - 3^2 + 4^2 - 5^2 + \cdots - 99^2$$

using the sigma notation. Then evaluate it.

(b) Repeat the same exercise as above for the sum

$$\frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{4}{16} + \cdots + \text{upto the first } n \text{ terms.}$$

2. Use mathematical induction to show that

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}.$$

3. Use Riemann sums to compute accurately (not estimate) the areas of the regions specified below:

(a) Below $y = x^2 + 2x + 3$, above $y = 0$, from $x = -1$ to $x = 2$.

(b) Above $x^2 - 2x$, below $y = 0$.

4. (a) If P_1 and P_2 are two partitions of $[a, b]$ such that every point of P_1 also belongs to P_2, then we say that P_2 is a refinement of P_1. Show that in this case

$$L(f, P_1) \leq L(f, P_2) \leq U(f, P_2) \leq U(f, P_1).$$

(b) Use the result above to show that every lower Riemann sum is less than or equal to every upper Riemann sum. This fact was critical in our defintion of the Riemann integral.

5. Using the interpretation of Riemann integral as area and using properties of the definite integral, evaluate:

(a) $\displaystyle\lim_{n\to\infty}\sum_{i=1}^{n}\frac{2n+3i}{n^2}.$

(b) $\displaystyle\int_{-3}^{3}(2+t)\sqrt{9-t^2}\,dt.$

(c) $\displaystyle\int_{-1}^{2}\operatorname{sgn}(x)\,dx,$ where $\operatorname{sgn}(x)=\begin{cases}+1 & \text{if } x>0,\\ -1 & \text{if } x<0.\end{cases}$

(d) $\displaystyle\int_{-3}^{4}\left(|x+1|-|x-1|+|x+2|\right)\,dx.$

(e) $\displaystyle\int_{0}^{3}\frac{x^2-x}{|x-1|}\,dx.$

(f) the constant k minimizing the integral $\displaystyle\int_a^b (f(x)-k)^2\,dx,$ where f is a continuous function on $[a,b]$, $a<b$.