Introduction to Harmonic Analysis - Math 541
Fall 2012

- ***Instructor: Malabika Pramanik***
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- ***Office hours: To be announced.***

• **Web page:** The course website is

http://www.math.ubc.ca/~malabika/teaching/ubc/fall12/math541/index.html

Homework assignments and all relevant course information (such as changes to office hours if any, or solutions to homework problems if needed) will be posted here.

• **Text:** There are no required textbooks. The following textbooks are recommended.

– *Lectures on Harmonic Analysis (2003), by T.H. Wolff*, AMS, ISBN: 978-0-8218-3449-7.

– *An Introduction to Harmonic Analysis (3rd edition), by Y. Katznelson*, Cambridge, ISBN: 978-0-521-54359-2.

– *Singular Integrals and Differentiability Properties of Functions (1970), by E. Stein*, Princeton University Press, ISBN: 0-691-08079-8.

– *Introduction to Fourier Analysis on Euclidean Spaces (1971), by E. Stein, and G. Weiss*, Princeton University Press, ISBN: 0-691-08078-X.

– *Harmonic Analysis: Real-variable Methods, Orthogonality and Oscillatory Integrals (1993), by E. Stein*, Princeton University Press, ISBN: 0-691-03216-5.

– *Classical and Modern Fourier Analysis, by L. Grafakos.*

• **Pre-requisites:** Math 507 (Measure theory and Integration), 508 (Complex Analysis), 510 (Functional Analysis) or their equivalents.

• **Course outline :** The core topics of the course are the following:

1. *Basic material concerning Fourier series, Fourier transform and Fourier inversion*
   - Fourier basis for $L^2(\mathbb{T})$
   - Convolution
   - Approximate identities
   - Temperate distributions
   - Some applications

2. *Convergence of Fourier series*
   - Decay of Fourier coefficients
   - Uniform convergence of Fourier series
   - Pointwise convergence and almost everywhere divergence
   - Norm convergence
3. *Interpolation of operators*
   - Complex methods (Riesz-Thörin theorem, analytic interpolation)
   - Real methods (Marcinkiewicz interpolation theorem)
   - Applications (Hausdorff-Young inequality, Young's convolution inequality, fractional integration, Hardy-Littlewood maximal theorem).
4. *Singular integral operators*
   - Calderón-Zygmund kernels
   - Some multiplier operators
   - The Calderón-Zygmund decomposition
   - L^p boundedness of Calderón-Zygmund singular integral operators
   - Homogeneous distributions, Hilbert transform, Riesz transform.
5. *Littlewood-Paley theory*
   - Almost orthogonality in Hilbert spaces, Cotlar-Knapp-Stein lemma
   - A square function that characterizes L^p
   - Variations and applications

Time permitting, we will also consider other special topics.

• **Lectures :** Monday, Wednesday, Friday 2 pm - 3 pm in Mathematics 202.

• **Grading Policy :** Your grade in this course will be based entirely on homework assignments, which will be posted regularly on the course website. You will be evaluated both on the mathematical rigour of your solutions as well as clarity of exposition, so please pay attention to details when preparing your answers. **Late homework assignments will not be accepted without prior consent of the instructor.**