Final - Math 440/508, Fall 2011

Due on Tuesday December 6

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1. Let n be a positive integer. Prove that the polynomial

$$f(x) = \sum_{i=0}^{n} \frac{x^i}{i!}$$

has n distinct complex zeroes $z_1, \cdots, z_n$, and they satisfy

$$\sum_{i=1}^{n} z_i^{-j} = 0 \quad \text{for} \quad 2 \le j \le n.$$

2. Let C denote the positively oriented circle $|z| = 2$. Evaluate the integral

$$\int_C \sqrt{z^2 - 1}\, dz$$

where the branch of the square root is chosen so that $\sqrt{2^2 - 1} > 0$.

3. Let Ω be the region whose boundaries are the rays $\operatorname{Arg}(z) = \pm\frac{\pi}{4}$ and the branch of the hyperbola $x^2 - y^2 = 1$ lying in the half-plane $\operatorname{Re}(z) > 0$. Find a conformal map of Ω onto the unit disk.