

Practice Problem Set 1 for Midterm 2

1. Find the volume above the paraboloid $z = x^2 + y^2$ and below the half-cone $z = \sqrt{x^2 + y^2}$.

(Answer: $\frac{\pi}{6}$)

2. Use the transformation $x = u^2$, $y = v^2$, $z = w^2$ to find the volume of the region bounded by the surface

$$\sqrt{x} + \sqrt{y} + \sqrt{z} = 1$$

and the coordinate hyperplanes.

(Answer: $\frac{1}{90}$)

3. Use spherical coordinates to evaluate

$$\int_{-2}^2 \int_0^{\sqrt{4-y^2}} \int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} y^2 \sqrt{x^2 + y^2 + z^2} dz dx dy.$$

(Answer: $\frac{64}{9}\pi$)

4. A lamp has three bulbs, each of a type with average lifetime 800 hours. If we model the probability of failure of bulbs by an exponential density function with mean 800, find the probability that all three bulbs fail within a total of 1000 hours.

(Answer: $1 - \frac{97}{32}e^{-\frac{5}{4}}$)

5. Describe the solid whose volume is given by the integral

$$\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \int_1^2 \rho^2 \sin \phi d\rho d\phi d\theta$$

and use this information (without any integration) to evaluate the integral.

(Answer: $\frac{7}{6}\pi$)