Math 217 Assignment 7

Due Friday November 6

■ **Problems to turn in:**

1. The boundary of a lamina consists of the semicircles $y = \sqrt{1-x^2}$ and $y = \sqrt{4-x^2}$ together with the portions of the x-axis that join them. Find the center of mass of the lamina if the density at any point is proportional to its distance from the origin.
2. (a) Evaluate the triple integral

$$\iiint_E xy dV,$$

where E is bounded by the parabolic cylinders $y = x^2$ and $x = y^2$ and the planes $z = 0$ and $z = x + y$.

(b) Evaluate the triple integral

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} xy\, dz\, dy\, dx.$$

(c) Use a triple integral to find the volume of the solid enclosed by the cylinder $x^2 + y^2 = 9$ and the planes $y + z = 5$ and $z = 1$.

(d) Find the mass and center of mass of the tetrahedron bounded by the planes $x = 0$, $y = 0$, $z = 0$, $x + y + z = 1$ with density function $\rho(x, y, z) = y$.

(e) Find the volume of the solid that lies within the both the cylinder $x^2 + y^2 = 1$ and the sphere $x^2 + y^2 + z^2 = 4$.
3. Identify the surface whose equation is given by $2r^2 + z^2 = 1$.
4. Suppose X, Y and Z are random variables with joint density function

$$f(x,y,z) = \begin{cases} Ce^{-(0.5x+0.2y+0.1z)} & \text{if } x, y, z \geq 0, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the value of the constant C.

(b) Find $P(X \leq 1, Y \leq 1)$.

(c) Find $P(X \leq 1, Y \leq 1, Z \leq 1)$.
5. Assume that Vancouver is a circular city of radius 10 miles in which the population is evenly distributed. In a study of the spread of the H1N1 virus in the city, a biologist assumes that the probability that an infected individual will spread the disease to an uninfected individual is a function of the distance between them; more specifically, for an uninfected individual residing at the point $A(x_0, y_0)$, the probability density of being infected by an individual at P is given by

$$f(P, A) = \frac{1}{20}(20 - d(P, A)),$$

where $d(P, A)$ denotes the distance between P and A.

The study works with the following two hypotheses. Suppose the exposure of a person to a disease is the sum of the probabilities of catching the disease from all members of the population; and that the infected people are uniformly distributed throughout the city, with k infected individuals per square mile.

(a) Find a double integral that represents the probability of exposure of a person residing at A. Do not evaluate this integral.

(b) Evaluate the integral for the case in which A is the center of the city and for the case in which A is located on the edge of the city. Where would you prefer to live?