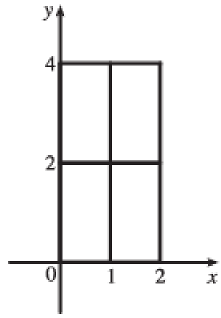
4. (a) The subrectangles are shown in the figure.



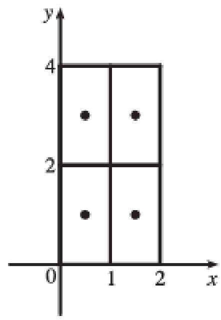
The surface is the graph of $f(x,y) = x + 2y^2$ and $\Delta A = 2$, so we estimate

$$V = \iint_R (x + 2y^2)\, dA \approx \sum_{i=1}^{2} \sum_{j=1}^{2} f\left(x_{ij}^*, y_{ij}^*\right) \Delta A$$

$$= f(1,0)\,\Delta A + f(1,2)\,\Delta A + f(2,0)\,\Delta A + f(2,2)\,\Delta A$$

$$= 1(2) + 9(2) + 2(2) + 10(2) = 44$$

(b) $V = \iint_R (x + 2y^2)\, dA \approx \sum_{i=1}^{2} \sum_{j=1}^{2} f(\overline{x}_i, \overline{y}_j)\, \Delta A$



$$= f\left(\tfrac{1}{2}, 1\right) \Delta A + f\left(\tfrac{1}{2}, 3\right) \Delta A + f\left(\tfrac{3}{2}, 1\right) \Delta A + f\left(\tfrac{3}{2}, 3\right) \Delta A$$

$$= \tfrac{5}{2}(2) + \tfrac{37}{2}(2) + \tfrac{7}{2}(2) + \tfrac{39}{2}(2) = 88$$

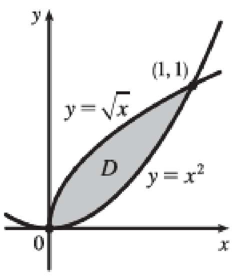
18. $$\iint_R \frac{1+x^2}{1+y^2}\, dA = \int_0^1 \int_0^1 \frac{1+x^2}{1+y^2}\, dy\, dx = \int_0^1 (1+x^2)\, dx \int_0^1 \frac{1}{1+y^2}\, dy = \left[x + \frac{1}{3}x^3\right]_0^1 \left[\tan^{-1} y\right]_0^1$$

$$= \left(1 + \tfrac{1}{3} - 0\right)\left(\tfrac{\pi}{4} - 0\right) = \tfrac{\pi}{3}$$

30. The cylinder intersects the xy-plane along the line $x = 4$, so in the first octant, the solid lies below the surface $z = 16 - x^2$ and above the rectangle $R = [0,4] \times [0,5]$ in the xy-plane.

$$V = \int_0^5 \int_0^4 (16 - x^2)\, dx\, dy = \int_0^4 (16 - x^2)\, dx \int_0^5 dy = \left[16x - \tfrac{1}{3}x^3\right]_0^4 \left[\, y\, \right]_0^5 = \left(64 - \tfrac{64}{3} - 0\right)(5 - 0) = \tfrac{640}{3}$$

14.

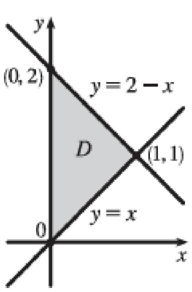


$$\int_0^1 \int_{x^2}^{\sqrt{x}} (x+y)\, dy\, dx = \int_0^1 \left[xy + \tfrac{1}{2}y^2\right]_{y=x^2}^{y=\sqrt{x}} dx$$

$$= \int_0^1 \left(x^{3/2} + \tfrac{1}{2}x - x^3 - \tfrac{1}{2}x^4\right) dx$$

$$= \left[\tfrac{2}{5}x^{5/2} + \tfrac{1}{4}x^2 - \tfrac{1}{4}x^4 - \tfrac{1}{10}x^5\right]_0^1 = \tfrac{3}{10}$$

24.

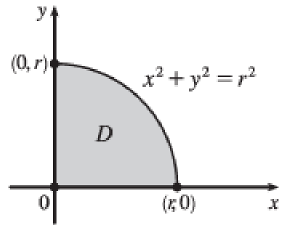


$$V = \int_0^1 \int_x^{2-x} x\, dy\, dx$$

$$= \int_0^1 x\left[\, y\, \right]_{y=x}^{y=2-x} dx = \int_0^1 (2x - 2x^2)\, dx$$

$$= \left[x^2 - \tfrac{2}{3}x^3\right]_0^1 = \tfrac{1}{3}$$

28.



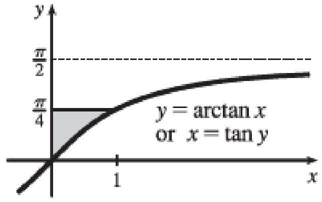
By symmetry, the desired volume V is 8 times the volume V_1 in the first octant. Now

$$V_1 = \int_0^r \int_0^{\sqrt{r^2-y^2}} \sqrt{r^2-y^2}\,dx\,dy = \int_0^r \left[x\sqrt{r^2-y^2}\right]_{x=0}^{x=\sqrt{r^2-y^2}} dy$$

$$= \int_0^r (r^2-y^2)\,dy = \left[r^2y - \tfrac{1}{3}y^3\right]_0^r = \tfrac{2}{3}r^3$$

Thus $V = \frac{16}{3}r^3$.

44.



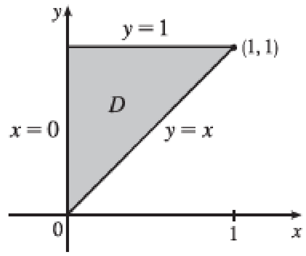
Because the region of integration is

$$D = \left\{(x,y) \mid \arctan x \le y \le \tfrac{\pi}{4}, 0 \le x \le 1\right\}$$
$$= \left\{(x,y) \mid 0 \le x \le \tan y, 0 \le y \le \tfrac{\pi}{4}\right\}$$

we have

$$\int_0^1 \int_{\arctan x}^{\pi/4} f(x,y)\,dy\,dx = \iint_D f(x,y)\,dA = \int_0^{\pi/4} \int_0^{\tan y} f(x,y)\,dx\,dy$$

48.



$$\int_0^1 \int_x^1 e^{x/y}\,dy\,dx = \int_0^1 \int_0^y e^{x/y}\,dx\,dy = \int_0^1 \left[ye^{x/y}\right]_{x=0}^{x=y} dy$$

$$= \int_0^1 (e-1)y\,dy = \tfrac{1}{2}(e-1)y^2\Big]_0^1$$

$$= \tfrac{1}{2}(e-1)$$

26. The two paraboloids intersect when $3x^2+3y^2 = 4-x^2-y^2$ or $x^2+y^2=1$. So

$$V = \iint\limits_{x^2+y^2\le 1} \left[(4-x^2-y^2) - 3(x^2+y^2)\right] dA = \int_0^{2\pi} \int_0^1 4(1-r^2)\,r\,dr\,d\theta$$

$$= \int_0^{2\pi} d\theta \int_0^1 (4r-4r^3)\,dr = \left[\theta\right]_0^{2\pi} \left[2r^2 - r^4\right]_0^1 = 2\pi$$

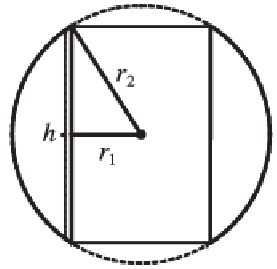
28. (a) Here the region in the xy-plane is the annular region $r_1^2 \le x^2 + y^2 \le r_2^2$ and the desired volume is twice that above the xy-plane. Hence

$$V = 2 \iint\limits_{r_1^2 \le x^2+y^2 \le r_2^2} \sqrt{r_2^2 - x^2 - y^2}\, dA = 2\int_0^{2\pi}\int_{r_1}^{r_2} \sqrt{r_2^2 - r^2}\, r\, dr\, d\theta = 2\int_0^{2\pi} d\theta \int_{r_1}^{r_2} \sqrt{r_2^2 - r^2}\, r\, dr$$
$$= \tfrac{4\pi}{3}\left[-(r_2^2 - r^2)^{3/2}\right]_{r_1}^{r_2} = \tfrac{4\pi}{3}(r_2^2 - r_1^2)^{3/2}$$

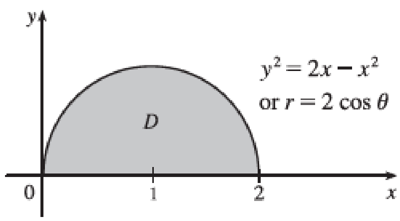
(b) A cross-sectional cut is shown in the figure.

So $r_2^2 = \left(\frac{1}{2}h\right)^2 + r_1^2$ or $\frac{1}{4}h^2 = r_2^2 - r_1^2$.

Thus the volume in terms of h is $V = \frac{4\pi}{3}\left(\frac{1}{4}h^2\right)^{3/2} = \frac{\pi}{6}h^3$.



32.



$$\int_0^{\pi/2}\int_0^{2\cos\theta} r^2\, dr\, d\theta = \int_0^{\pi/2}\left[\tfrac{1}{3}r^3\right]_{r=0}^{r=2\cos\theta} d\theta = \int_0^{\pi/2}\left(\tfrac{8}{3}\cos^3\theta\right) d\theta$$
$$= \tfrac{8}{3}\int_0^{\pi/2}(1-\sin^2\theta)\cos\theta\, d\theta$$
$$= \tfrac{8}{3}\left[\sin\theta - \tfrac{1}{3}\sin^3\theta\right]_0^{\pi/2} = \tfrac{16}{9}$$