Math 263, Fall 2008
Midterm 2, November 12

Name: **SID:**

Instructor: Pramanik **Section: 102**

Instructions

- The total time is 50 minutes.
- The total score is 100 points.
- Use the reverse side of each page if you need extra space.
- Show all your work. A correct answer without intermediate steps will receive no credit.
- Calculators and cheat sheets are not allowed.

Problem	**Points**	**Score**
1	15	
2	25	
3	20	
4	20	
5	20	
TOTAL	100	

1. Let $\mathbf{F}(x, y, z) = (\sin x, 2\cos x, 1 - y^2)$.

(a) Calculate curl $\mathbf{F}$.

(5 points)

(b) Calculate div $\mathbf{F}$.

(5 points)

(c) Calculate div(curl $\mathbf{F}$).

(5 points)

2. Sketch the domain of integration for the integral given below. Then convert the integral to spherical coordinates and evaluate it.

$$\int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_{0}^{\sqrt{9-x^2-y^2}} z\sqrt{x^2+y^2+z^2} \quad dzdydx$$

(5 + 10 + 10 = 25 points)

3. Is the vector field $\mathbf{F}(x, y, z) = (2xy+y^2)\mathbf{i}+(x^2+2xy+z^2)\mathbf{j}+2zy\mathbf{k}$ conservative? If so, find a function f so that $\mathbf{F} = \nabla f$. If not, explain clearly why.

(20 points)

4. Find the line integral of $\mathbf{F}(x, y, z) = (yz)\mathbf{i} + (xz)\mathbf{j} + (xy + 1)\mathbf{k}$ around the square with corners at $(0, 0, 1), (1, 0, 1), (1, 1, 1)$ and $(0, 1, 1)$ (taken in that order).

(20 points)

5. (a) State Green's theorem for $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is a simple, positively oriented, closed curve in the (x, y) plane and $\mathbf{F}(x, y) = P(x, y)\mathbf{i} + Q(x, y)\mathbf{j}$ is a two dimensional vector field.

(4 points)

(b) Compute the work done by the force field $\mathbf{F}(x, y) = \mathbf{i} + x\mathbf{j}$ on a particle that makes one counterclockwise revolution around the circle $x^2 + y^2 = 1$.

(8 points)

(c) Compute the work done by the force field $\mathbf{F}(x, y) = \mathbf{i} + x\mathbf{j}$ on a particle that travels from $(1, 0)$ to $(0, 1)$, counterclockwise along part of the circle $x^2 + y^2 = 1$.

(8 points)