Math 263, Fall 2008
Midterm I, October 15

Name: **SID:**

Instructor: Pramanik **Section: 102**

Instructions

- The total time is 50 minutes.
- The total score is 100 points.
- Use the reverse side of each page if you need extra space.
- Show all your work. A correct answer without intermediate steps will receive no credit.
- Calculators and cheat sheets are not allowed.

Problem	Points	Score
1	15	
2	15	
3	15	
4	20	
5	20	
6	15	
TOTAL	100	

1. A rectangular box lies in the region $x \geq 0, y \geq 0, z \geq 0$. One corner of the box is at $(0,0,0)$ and another corner lies on the sphere $x^2 + y^2 + z^2 = 3$. Find the maximum possible volume of the box. (15 points)

2. Find all the points on the surface $x^2 + 4y^2 - z^2 = 4$ at which the normal direction to the surface is perpendicular to the plane $2x + 2y + z = 5$.

(15 points)

3. The water temperature in a swimming pool is given by the equation

$$T = x^2 e^y - xy^3.$$

Suppose you swim through the point $(1, 0)$. What is the maximum possible rate of increase in water temperature that you could experience as you swim through that point? In which direction should you swim in order to feel this maximum rate of increase in temperature?

(7 + 8 = 15 points)

4. Sketch the domain of integration and evaluate

$$\int_0^4 \int_{\sqrt{x}}^2 \frac{1}{y^3+1}\, dydx.$$

(20 points)

5. (a) Can there be a continuously differentiable function f satisfying

$$f_x = 2xy^2 \quad \text{and} \quad f_y = 2x^2y?$$

If so, find such a function. If not, explain clearly why.

(10 points)

(b) Can there be a continuously differentiable function f satisfying

$$f_x = 2x^2y \quad \text{and} \quad f_y = 2xy^2?$$

If so, find such a function. If not, explain clearly why.

(10 points)

6. Let C be the curve of intersection of the surface $z = 2x^2 - y^2$ and the plane $z = 4$. Find parametric equations for the tangent line to C at the point $(-2, 2, 4)$.

(15 points)