Math 263 Assignment 1

Due September 12

Problems from the text (do NOT turn in these problems):

(13.1) 11-14, 15, 17, 40; (13.2) 17-22, 29, 32, 33; (13.3) 7-10, 11, 12, 15, 17, 49, 50; (13.4) 1-4, 29, 30; (13.5) 2-14, 23-38, 46, 48, 51, 52, 56, 61, 65.

Problems to turn in:

1) Find the equation of a sphere if one of its diameters has end points $(2, 1, 4)$ and $(4, 3, 10)$.

2) Show that the set of all points P that are twice as far from $(-1, 5, 3)$ as from $(6, 2, -2)$ is a sphere. Find its centre and radius.

3) Describe and sketch the set of all points in $\mathbb{R}^3$ that satisfy

   a) $x^2 + y^2 + z^2 = 2z$ b) $x^2 + z^2 = 4$ c) $z \geq \sqrt{x^2 + y^2}$

   d) $x^2 + y^2 + z^2 = 4,\ z = 1$ e) $x + y + z = 1$

4) Compute the dot product of the vectors $\vec{a}$ and $\vec{b}$. Find the angle between them.

   a) $\vec{a} = \langle -1, 1 \rangle,\ \vec{b} = \langle 1, 1 \rangle$ b) $\vec{a} = \langle 1, 1 \rangle,\ \vec{b} = \langle 2, 2 \rangle$

5) Use a projection to derive a formula for the distance from a point (x_1, y_1) to the line $ax + by = c$. Here, a and b are not both zero.

6) Compute $\langle 1, 2, 3 \rangle \times \langle 4, 5, 6 \rangle$.

7) Prove that

   a) $\hat{\boldsymbol{\imath}} \times \hat{\boldsymbol{\jmath}} = \hat{\mathbf{k}}$

   b) $\vec{a} \cdot (\vec{a} \times \vec{b}) = \vec{b} \cdot (\vec{a} \times \vec{b}) = 0$

   c) $|\vec{a} \times \vec{b}|^2 = |\vec{a}|^2 \, |\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2$

8) Find the equation of the sphere which has the two planes $x+y+z = 3,\ x+y+z = 9$ as tangent planes if the centre of the sphere is on the planes $2x - y = 0,\ 3x - z = 0$.

9) Find the equation of the plane that passes through the point $(-2, 0, -1)$ and through the line of intersection of $2x + 3y - z = 0,\ x - 4y + 2z = -5$.

10) Find the equation of the line through $(2, -1, -1)$ and parallel to each of the two planes $x + y = 0$ and $x - y + 2z = 0$.