Math 100:V02 – WORKSHEET 8
APPLICATIONS OF THE CHAIN RULE

1. Review

(1) Differentiate
  (a) $e^{\sqrt{\cos x}}$

(2) (Final, 2014) Let $y = x^{\log x}$. Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x only.

2. Implicit Differentiation

(3) Find the line tangent to the curve $y^2 = 4x^3 + 2x$ at the point $(2, 6)$.

(4) (Final, 2015) Let $xy^2 + x^2y = 2$. Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the point $(1, 1)$.

(5) (Final 2012) Find the slope of the line tangent to the curve $y + x\cos y = \cos x$ at the point $(0, 1)$.

(6) Find y'' (in terms of x, y) along the curve $x^5 + y^5 = 10$ (ignore points where $y = 0$).

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3. Related Rates

(5) A particle is moving along the curve $y^2 = x^3 + 2x$. When it passes the point $\left(1, \sqrt{3}\right)$ we have $\frac{\mathrm{d}y}{\mathrm{d}t} = 1$. Find $\frac{\mathrm{d}x}{\mathrm{d}t}$.

(6) The state of a quantity of gas in a piston must satisfy the *ideal gas law*

$$PV = nRT\,,$$

where P is the pressure, V is the volume, n is the number of moles of gas, T is the (absolute) temperature and R is the ideal gas constant. Suppose $P = 1\text{atm}$ and $V = 22.4\text{L}$. How fast is the pressure of the gas changing when $\frac{dV}{dt} = 2.5\frac{\text{L}}{\text{min}}$, if the expansion is *isothermal*, that is with T held constant?

4. Partial derivatives

(7) Returning to the equation $PV = nRT$ now treat the temperature as a *function* of both pressure and volume.

(a) Suppose the volume is constant. What is the rate of change of temperature with respect to pressure?

(b) Suppose the pressure is constant. What is the rate of change of temperature with respect to pressure?

(c) What is the rate of change of the temperaure with respet to the number of moles of gas, pressure and volume being constant?