Math 100 – WORKSHEET 14
TAYLOR EXPANSION

1. Taylor approximation

(1) (Review) Use linear approximations to estimate:
   (a) $\log \frac{4}{3}$ and $\log \frac{2}{3}$. Combine the two for an estimate of $\log 2$.

   (b) $\sin 0.1$ and $\cos 0.1$.

(2) Let $f(x) = e^x$
   (a) Find $f(0), f'(0), f^{(2)}(0), \cdots$
   (b) Find a polynomial $T_0(x)$ such that $T_0(0) = f(0)$.
   (c) Find a polynomial $T_1(x)$ such that $T_1(0) = f(0)$ and $T_1'(0) = f'(0)$.
   (d) Find a polynomial $T_2(x)$ such that $T_2(0) = f(0)$, $T_2'(0) = f'(0)$ and $T_2^{(2)}(0) = f^{(2)}(0)$.
   (e) Find a polynomial $T_3(x)$ such that $T_3^{(k)}(0) = f^{(k)}(0)$ for $0 \leq k \leq 3$.

(3) Do the same with $f(x) = \ln x$ about $x = 1$.

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Let $c_k = \frac{f^{(k)}(a)}{k!}$. The nth order Taylor expansion of $f(x)$ about $x = a$ is the polynomial

$$T_n(x) = c_0 + c_1(x-a) + \cdots + c_n(x-a)^n$$

(4) Find the 4th order MacLaurin expansion of $\frac{1}{1-x}$ (=Taylor expansion about $x = 0$)

(5) Find the nth order expansion of $\cos x$, and approximate $\cos 0.1$ using a 3rd order expansion

(6) (Final, 2015) Let $T_3(x) = 24 + 6(x-3) + 12(x-3)^2 + 4(x-3)^3$ be the third-degree Taylor polynomial of some function f, expanded about $a = 3$. What is $f''(3)$?

2. New from old

(7) (Final, 2016) Use a 3rd order Taylor approximation to estimate $\sin 0.01$. Then find the 3rd order Taylor expansion of $(x+1)\sin x$ about $x = 0$.

(8) Find the 3rd order Taylor expansion of $\sqrt{x} + 3x$ about $x = 4$.

(9) Find the 8th order expansion of $f(x) = e^{x^2} + \cos(2x)$. What is $f^{(6)}(0)$?

(10) Show that $\log \frac{1+x}{1-x} \approx 2(x + \frac{x^3}{3} + \frac{x^5}{5} + \cdots)$. Use this to get a good approximation to $\log 3$ via a careful choice of x.