Math 100 – WORKSHEET 17
THE MEAN VALUE THEOREM; LINEAR APPROXIMATION

1. Average slope vs Instantenous slope

(1) Let $f(x) = e^x$ on the interval $[0, 1]$. Find all values of c so that $f'(c) = \frac{f(1)-f(0)}{1-0}$.

(2) Let $f(x) = |x|$ on the interval $[-1, 2]$. Find all values of c so that $f'(c) = \frac{f(2)-f(-1)}{2-(-1)}$

2. The Mean Value Theorem

Theorem. *Let f be defined and differentiable on $[a, b]$. Then there is c between a, b such that $\frac{f(b)-f(a)}{b-a} = f'(c)$.*
Equivalently, for any x there is c between a, x so that $f(x) = f(a) + f'(c)(x - a)$.

(3) Show that $f(x) = 3x^3 + 2x - 1 + \sin x$ has exactly one real zero. (Hint: let a, b be zeroes of f. The MVT will find c such that $f'(c) =$?)

(4) (Final, 2015)
 (a) Suppose f, f', f'' are all continuous. Suppose f has at least three zeroes. How many zeroes must f', f'' have?

 (b) [Show that $2x^2 - 3 + \sin x + \cos x = 0$ has at least two solutions]
 (c) Show that the equation has at most two solutions.

(5) (Final, 2012) Suppose $f(1) = 3$ and $-3 \leq f'(x) \leq 2$ for $x \in [1, 4]$. What can you say about $f(4)$?

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(6) Show that $|\sin a - \sin b| \leq |a - b|$ for all a, b.

(7) Let $x > 0$. Show that $e^x > 1 + x$ and that $\log(1 + x) < x$.

3. The Linear Approximation

Fact: For x near a we have $f(x) \approx L(x)$ where $\boxed{L(x) = f(a) + f'(a)(x - a)}$

(8) Use a linear approximation to estimate

(a) $\sqrt{1.2}$

(b) (Final, 2015) $\sqrt{8}$

(c) (Final, 2016) $(26)^{1/3}$

(d) $\log 1.07$