

Math 100 – WORKSHEET 11
INVERSE TRIG FUNCTIONS; RELATED RATES

1. INVERSE TRIG FUNCTIONS

(1) Evaluation

(a) (Final 2014) Evaluate $\arcsin\left(-\frac{1}{2}\right)$; Find $\arcsin\left(\sin\left(\frac{31\pi}{11}\right)\right)$.

(b) (Final 2015) Simplify $\sin(\arctan 4)$

(c) Find $\tan(\arccos(0.4))$

(2) Differentiation

(a) Find $\frac{d}{dx}(\arcsin(2x))$

(b) Find the line tangent to $y = \sqrt{1 + (\arctan(x))^2}$ at the point where $x = 1$.

(c) Find y' if $y = \arcsin(e^{5x})$. What is the domain of the functions y, y' ?

2. VELOCITY AND ACCELERATION

An object moves by $s = f(t)$. Then the *velocity* is $v(t) = \frac{ds}{dt}$ and the *acceleration* is $a(t) = \frac{dv}{dt} = \frac{d^2s}{dt^2}$.

- (3) A particle's position is given by $f(t) = t + 6e^{-t/3}$.
- (a) Find the velocity at time t , and specifically at $t = 3$.

 - (b) When is the particle moving to the right? to the left?

 - (c) When is the particle accelerating? decelerating?
- (4) (Final, 2016) An object is thrown straight up into the air at time $t = 0$ seconds. Its height in metres at time t seconds is given by $h(t) = s_0 + v_0t - 5t^2$. In the first second the object rises by 5 metres. For how many seconds does the object rise before beginning to fall?
- (5) A emergency braking car can decelerate at $9\frac{\text{m}}{\text{s}^2}$. How fast can a car drive so that it can come to a stop within 50m?

3. RELATED RATES

- (6) A particle is moving along the curve $y^2 = x^3 + 2x$. When it passes the point $(1, \sqrt{3})$ we have $\frac{dy}{dt} = 1$. Find $\frac{dx}{dt}$.
- (7) (Final, 2015, variant) A conical tank of water is 6m tall and has radius 1m at the top.
- (a) The drain is clogged, and is filling up with rainwater at the rate of $5\text{m}^3/\text{min}$. How fast is the water rising when its height is 5m?

 - (b) The drain is unclogged and water begins to drain at the rate of $(5 + \frac{\pi}{4})\text{m}^3/\text{min}$ (but rain is still falling). At what height is the water falling at the rate of $1\text{m}/\text{min}$?