

**Math 100 – WORKSHEET 7**  
**DIFFERENTIATION RULES**

1. THE PRODUCT AND QUOTIENT RULES

**Fact.**  $(af + bg)' = af' + bg'$ ,  $(fg)' = f'g + fg'$ ,  $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$

(1) Differentiate

(a)  $f(x) = 6x^\pi + 2x^e - x^{7/2}$

(b) (Final, 2016)  $g(x) = x^2e^x$  (and then also  $x^ae^x$ )

(c) (Final, 2016)  $h(x) = \frac{x^2+3}{2x-1}$

(d)  $\frac{x^2+A}{\sqrt{x}}$

(2) Let  $f(x) = \frac{x}{\sqrt{x+A}}$ . Given that  $f'(4) = \frac{3}{16}$ , give a quadratic equation for  $A$ .

- (3) Suppose that  $f(1) = 1$ ,  $g(1) = 2$ ,  $f'(1) = 3$ ,  $g'(1) = 4$ . Find  $(fg)'(1)$  and  $\left(\frac{f}{g}\right)'(1)$ .

## 2. THE TANGENT LINE

**Definition.** The line tangent to the graph  $y = f(x)$  at  $x = a$  is the line  $y = f'(a)(x - a) + f(a)$

- (1) (Final, 2015) Find the equation of the line tangent to the function  $f(x) = \sqrt{x}$  at  $(4, 2)$ .
- (2) Let  $f(x) = \frac{g(x)}{x}$ , where  $g(x)$  is differentiable at  $x = 1$ . The line  $y = 2x - 1$  is tangent to the graph  $y = f(x)$  at  $x = 1$ . Find  $g(1)$  and  $g'(1)$ .
- (3) (Final 2015) The line  $y = 4x + 2$  is tangent at  $x = 1$  to which function:  $x^3 + 2x^2 + 3x$ ,  $x^2 + 3x + 2$ ,  $2\sqrt{x+3} + 2$ ,  $x^3 + x^2 - x$ ,  $x^3 + x + 2$ , none of the above?

- (4) Find the lines of slope 3 tangent the curve  $y = x^3 + 4x^2 - 8x + 3$ .

- (5) The line  $y = 5x + B$  is tangent to the curve  $y = x^3 + 2x$ . What is  $B$ ?