Math 100 – WORKSHEET 6
THE DERIVATIVE

1. DEFINITION OF THE DERIVATIVE

Definition. $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

(1) Find $f'(a)$ if
  (a) $f(x) = x^2$, $a = 3$.

  (b) $f(x) = \frac{1}{x}$, any a.

  (c) $f(x) = x^3 - 2x$, any a (you may use $(a+h)^3 = a^3 + 3a^2h + 3ah^2 + h^3$).

(2) Express the limits as derivatives: $\lim_{h \to 0} \frac{\cos(5+h) - \cos 5}{h}$, $\lim_{h \to 0} \frac{\sin x}{x}$

(3) (Final, 2015) Is the function
$$f(x) = \begin{cases} \sqrt{1 + x^2} - 1 & x \leq 0 \\ x^2 \cos \frac{1}{x} & x > 0 \end{cases}$$
differentiable at $x = 0$?

2. Linear combinations; power laws

(1) If f, g are functions and a, b are numbers then $(af + bg)' = af' + bg'$

(2) $\frac{d}{dx}(x^r) = rx^{r-1}$

(4) Let $g(y) = Ay^{5/2} + y^2$. Suppose that $g'(4) = 0$. What is A?

(5) Find the *second* derivative of $5t + 3\sqrt{t}$

(6) Differentiate $f(x) = \frac{5x^3 - 2x + 1}{\sqrt{x}}$.

(7) (Final, 2015) Find the equation of the line tangent to the function $f(x) = \sqrt{x}$ at $(4, 2)$.