Math 100 – WORKSHEET 2
LIMIT LAWS

1. Existence of limits and blowup

(1) Either evaluate the limit or explain why it does not exist. Sketching a graph might be helpful.

(a) $\lim_{x\to 1} f(x)$ where $f(x) = \begin{cases} \sqrt{x} & 0 \le x < 1 \\ 3 & x = 1 \\ 2 - x^2 & x > 1 \end{cases}$.

(b) $\lim_{x\to 1} f(x)$ where $f(x) = \begin{cases} \sqrt{x} & 0 \le x < 1 \\ 1 & x = 1 \\ 4 - x^2 & x > 1 \end{cases}$.

(2) Let $f(x) = \frac{x-3}{x^2+x-12}$.

(a) (Final 2014) What is $\lim_{x\to 3} f(x)$?

(b) What about $\lim_{x\to 2} f(x)$?

Date: 14/9/2021, Worksheet by Lior Silberman. This instructional material is excluded from the terms of UBC Policy 81.

2. Limit Laws

Fact. *Limits respect arithmetic operations and standard functions (e^x, sin, cos, log, ...)* **as long as everything is well-defined.**

(beware especially of division by zero)

(3) Evaluate using the limit laws:

(a) $\lim_{x\to 2} \frac{x+1}{4x^2-1} =$

(b) $\lim_{x\to 1} \frac{e^x(x-1)}{x^2+x-2} =$

(4) Evaluate using the identity $\sqrt{a}-\sqrt{b} = \left(\sqrt{a}-\sqrt{b}\right)\cdot\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}+\sqrt{b}} = \frac{a-b}{\sqrt{a}+\sqrt{b}}$:

(a) $\lim_{x\to 0} \frac{\sqrt{4+x}-2}{x}$.

(b) $\lim_{x\to 0} \frac{\sqrt{1+x}-\sqrt{1+x^2}}{x}$.

(5) Evaluate using the Sandwich/Squeeze Theorem

(a) $\lim_{x\to 0} x^2 \sin\left(\frac{\pi}{x}\right)$.

(b) (Final, 2014) Suppose that $8x \le f(x) \le x^2+16$ for all $x \ge 0$. Find $\lim_{x\to 4} f(x)$.