Math 100 – SOLUTIONS TO WORKSHEET 16
MINIMA AND MAXIMA

1. Absolute minima and maxima by hand

Theorem. *If f is continuous on $[a,b]$ it has an absolute maximum and minimum there.*

(1) Find the absolute maximum and minimum values of $f(x) = |x|$ on the interval $[-3, 5]$.
(2) Find the absolute maximum and minimum of $f(x) = \sqrt{x}$ on $[0, 5]$.

2. Derivatives and local extrema

Theorem (Fermat)**.** *If, in addition, f is defined and differentiable near c (on both sides!) and has a local extremum at c then $f'(c) = 0$.*

Procedure

- Call c a *critical point* if $f'(c) = 0$, a *singular point* if $f'(c)$ does not exist.
- To find absolute maximum/minimum of a continuous function f defined on $[a, b]$:
  - Evaluate $f(c)$ at all critical and singular point.
  - Evaluate $f(a), f(b)$.
  - Choose largest, smallest value.

(3) (Final, 2011) Let $f(x) = 6x^{1/5} + x^{6/5}$.
   (a) Find the critical numbers and singularities of f.
   (b) Find its absolute maximum and minimum on the internal $[-32, 32]$.

(4) (Final, 2015) Find the critical points of $f(x) = e^{x^3-9x^2+15x-1}$
   Solution: By the chain rule

$$\begin{aligned} f'(x) &= f(x) \cdot \left(3x^2 - 18x + 15\right) \\ &= 3f(x)\left(x^2 - 6x + 5\right) \\ &= 3f(x)\,(x-5)\,(x-1)\ . \end{aligned}$$

   Since $e^y \neq 0$ for all y, f is never zero and thus $f'(x) = 0$ iff $(x-5)(x-1) = 0$ that is iff $x = 5$ or $x = 1$ and the critical points are $1, 5$.

(5) (caution)
   (a) Show that $f(x) = (x-1)^4 + 7$ attains its absolute minimum at $x = 1$.
   (b) Show that $f(x) = (x-1)^3 + 7$ has $f'(1) = 0$ but has no local minimum or maximum there.

(6) (Midterm, 2010) Find the maximum value of $x\sqrt{1 - \frac{3}{4}x^2}$ on the interval $[0, 1]$.

(7) (Final, 2007) Let $f(x) = x\sqrt{3-x}$.
   (a) Find the domain of f.
   (b) Determine the x-coordinates of any local maxima or minima of f.

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