# Math 100 – WORKSHEET 12
## EXPONENTIAL GROWTH AND DECAY

### 1. Exponentials

Growth/decay described by the *differential equation*

$$\frac{\mathrm{d}y}{\mathrm{d}t} = ky\,,$$

**Solution**: $y =$

(1) Suppose[1] that a pair of invasive opossums arrives in BC in 1935. Unchecked, opossums can triple their population annually.
   (a) At what time will there be 1000 opossums in BC? 10,000 opossums?

   (b) Write a differential equation expressing the growth of the opossum population with time.

(2) A radioactive sample decays according to the law

$$\frac{\mathrm{d}m}{\mathrm{d}t} = km\,.$$

   (a) Suppose that one-quarter of the sample remains after 10 hours. What is the half-life?
   (b) A 100-gram sample is left unattended for three days. How much of it remains?

(3) (Final, 2015) A colony of bacteria doubles every 4 hours. If the colony has 2000 cells after 6 hours, how many were present initially? Simplify your answer.

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[1]See http://linnet.geog.ubc.ca/efauna/Atlas/Atlas.aspx?sciname=Didelphis%20virginiana

## 2. Newton's Law of Cooling

**Fact.** *When a body of temperature $T_0$ is placed in an environment of temperature $T_{env}$, the rate of change of the temperature $T(t)$ is negatively proportional to the temperature difference $T - T_{env}$. In other words, there is a (negative) constant $k$ such that*

$$T' = k(T - T_{env}).$$

- *key idea*: change variables to the *temperature difference*. Let $y = T - T_{env}$. Then
$$\frac{dy}{dt} = \frac{dT}{dt} - 0 = ky$$

so there is $C$ for which
$$y(t) = Ce^{kt}.$$

Solving for $T$ we get:
$$T(t) = T_{env} + Ce^{kt}.$$

Setting $t = 0$ we find $T_{env} + C = T_0$ so $C = T_0 - T_{env}$ and
$$T(t) = T_{env} + (T_0 - T_{env})e^{kt}.$$

**Corollary.** $\lim_{t\to\infty} y(t) = 0$ *so* $\lim_{t\to\infty} T(y) = T_{env}$.

(1) (Final, 2010) When an apple is taken from a refrigerator, its temperature is $3°C$. After 30 minutes in a $19°C$ room its temperature is $11°C$.
   (a) Find the temperature of the apple 90 minutes after it is taken from the refrigerator, expressed as an integer number of degrees Celsius.

   (b) Determine the time when the temperature of the apple is $16°C$.

   (c) Write the *differential equation* satisfied by the temperature $T(t)$ of the apple.

(2) (Final, 2013) A bottle of soda pop at room temperature $(70°F)$ is placed in the refrigerator where the temperature is $40°F$. After half han hour the bottle has cooled to $60°F$. When will it reach $50°F$?