Math 100 – SOLUTIONS TO WORKSHEET 5
THE DERIVATIVE

1. Linear combinations; power laws

(1) If f,g are functions and a,b are numbers then $(af+bg)'=af'+bg'$ (2) $\frac{\mathrm{d}}{\mathrm{d}x}(x^r)=rx^{r-1}$ (3) $\frac{\mathrm{d}}{\mathrm{d}x}(e^x)=e^x$.

(1)
 (a) Differentiate $f(x)=\frac{5x^3-2x+1}{\sqrt{x}}$.
 Solution: Write $f(x)=5x^{5/2}-2x^{1/2}+x^{-1/2}$ and then $f'(x)=\frac{25}{2}x^{3/2}-x^{-1/2}-\frac{1}{2}x^{-3/2}$.
 (b) Let $g(x)=Ax^{5/2}+x^2$. Suppose that $g'(4)=0$. What is A?
 Solution: Differentiating we find $g'(x)=\frac{5}{2}Ax^{3/2}+2x$, so $0=g'(4)=\frac{5}{2}A\cdot 4^{3/2}+2\cdot 4=\frac{5}{2}\cdot A\cdot 8+8$. This means: $20A+8=0$ so $A=-\frac{2}{5}$.

(2) Find the *second* derivative of
 (a) $5e^x$
 (b) $\sqrt{x}+5e^x$
 Solution: $\frac{d}{dx}(5e^x)=5\frac{d}{dx}(e^x)=5e^x$ so the second derivative is also the same. Also, $(\sqrt{x})''=\left(\frac{1}{2}x^{-1/2}\right)'=-\frac{1}{4}x^{-3/2}$ so by linearity the second derivative of $\sqrt{x}+5e^x$ is $5e^x-\frac{1}{4x^{3/2}}$.

(3) The line $y=5x+B$ is tangent to the curve $y=x^3+2x$. What is B?
 Solution: At the point (x,y) the curve has slope $\frac{dy}{dx}=3x^2+2$, so the curve has slope 5 at the points where $x=\pm 1$, that is the points $(-1,-3)$ and $(1,3)$. The line needs to meet the curve at the point, so there are two solutions:

$$y=5x+2 \qquad \text{(tangent at } (-1,-3))$$
$$y=5x-2 \qquad \text{(tangent at } (1,3))$$

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2. The product and quotient rules

Fact. $(fg)' = f'g + fg'$, $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$

(1) Find $\frac{\mathrm{d}}{\mathrm{d}x}\left(x^a e^x\right)$.

Solution: We have $\frac{\mathrm{d}}{\mathrm{d}x}\left(x^a e^x\right) = \left(\frac{\mathrm{d}}{\mathrm{d}x} x^a\right) e^x + x^a \left(\frac{\mathrm{d}}{\mathrm{d}x} e^x\right) = a x^{a-1} e^x + x^a e^x$.

(2) Suppose that $f(1) = 1$, $g(1) = 2$, $f'(1) = 3$, $g'(1) = 4$. Find $(fg)'(1)$ and $\left(\frac{f}{g}\right)'(1)$.

Solution: $(fg)'(1) = f'(1)g(1) + f(1)g'(1) = 3 \cdot 2 + 1 \cdot 4 = 10$.

$$\left(\frac{f}{g}\right)'(1) = \frac{f'(1)g(1) - f(1)g'(1)}{(g(1))^2} = \frac{3 \cdot 2 - 1 \cdot 4}{2^2} = \frac{1}{2}\,.$$

(3) $f(x) = \frac{x^2 + A}{\sqrt{x}}$. $f'(x) =$

Solution: $f(x) = x^{3/2} + A x^{-1/2}$ so $f'(x) = \frac{3}{2} x^{1/2} - \frac{A}{2} x^{-3/2}$.

(4) Let $f(x) = \frac{x}{\sqrt{x} + A}$. Given that $f'(4) = \frac{3}{16}$, give a quadratic equation for A.

Solution: $f'(x) = \frac{1 \cdot \left(\sqrt{x} + A\right) - x\left(\frac{1}{2} x^{-1/2}\right)}{\left(\sqrt{x} + A\right)^2} = \frac{\sqrt{x} + A - \frac{1}{2}\sqrt{x}}{\left(\sqrt{x} + A\right)^2} = \frac{\frac{1}{2}\sqrt{x} + A}{\left(\sqrt{x} + A\right)^2}$. Plugging in $x = 4$ we have

$$\frac{3}{16} = f'(4) = \frac{1 + A}{(2 + A)^2}$$

so we have

$$3(2 + A)^2 = 16(1 + A)$$

that is

$$3A^2 + 12A + 12 = 16 + 16A$$

that is

$$3A^2 - 4A - 4 = 0\,.$$

In fact this gives $A = -\frac{2}{3}, 2$.