Math 100 – WORKSHEET 3
LIMITS AT INFINITY; CONTINUITY

1. The Squeeze Theorem

(1) $\lim_{x\to 0} x^2 \sin\left(\frac{\pi}{x}\right)$.

(2) (Final, 2014) Suppose that $8x \le f(x) \le x^2 + 16$ for all $x \ge 0$. Find $\lim_{x\to 4} f(x)$.

2. Limits at infinity

(1) Evaluate the following limits:
   (a) $\lim_{x\to\infty} \frac{x^2+1}{x-3} =$

   (b) (Final, 2015) $\lim_{x\to\infty} \frac{x+1}{x^2+2x-8} =$

   (c) (Quiz, 2015) $\lim_{x\to-\infty} \frac{3x}{\sqrt{4x^2+x}-2x} =$

   (d) $\lim_{x\to\infty} \frac{\sqrt{x^4+\sin x}}{x^2-\cos x} =$

   (e) $\lim_{x\to-\infty} \left(\sqrt{x^2+2x} - \sqrt{x^2-1}\right) =$

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3. Continuity

(1) Which of these functions are continuous everywhere? Why?

(a) $f(x) = \begin{cases} x & x < 0 \\ \cos x & x \geq 0 \end{cases}$

(b) $f(x) = \begin{cases} x & x < 0 \\ \sin x & x \geq 0 \end{cases}$

(2) Let $f(x) = \frac{x^3 - x^2}{x-1}$.

(a) Why is $f(x)$ discontinuous at $x = 1$?

(b) Find b such that $g(x) = \begin{cases} f(x) & x \neq 1 \\ b & x = 1 \end{cases}$ is continouous everywhere.

(c) Find c, d such that $f(x) = \begin{cases} \sqrt{x} & 0 \leq x < 1 \\ c & x = 1 \\ d - x^2 & x > 1 \end{cases}$ is continuous.

(d) (Final 2013) For which value of the constant c is $f(x) = \begin{cases} cx^2 + 3 & x \geq 1 \\ 2x^3 - c & x < 1 \end{cases}$ continuous on $(-\infty, \infty)$?

(3) Where are the following functions continuous?

(a) $\frac{1}{\sqrt{7-x^2}}$

(b) $\frac{x^2+2x+1}{2+\cos x}$

(c) $\frac{2+\cos x}{x^2+2x+1}$

(d) $\log(\sin x)$

(4) (Final 2011) Suppose f, g are continuous such that $g(3) = 2$ and $\lim_{x\to 3}(xf(x) + g(x)) = 1$. Find $f(3)$.