Math 101 – SOLUTIONS TO WORKSHEET 7
AREA BETWEEN CURVES

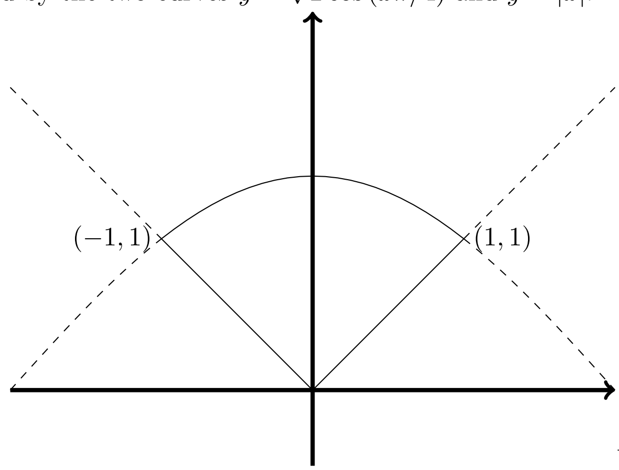
(1) Find the total area of the following planar regions. It will be useful to sketch the region first.

(a) (Final, 2011) The finite region lying between the curves $y = x$ and $y = x^3$.

Solution: The curves intersect where $x = x^3$, that is where $x(x+1)(x-1) = 0$. On $[-1, 0]$ we have $x \le x^3 \le 0$. On $[0, 1]$ we have $0 \le x^3 \le x$. By symmetry the areas are equal, and the total area is therefore

$$\int_{-1}^{0} (x^3 - x)\,\mathrm{d}x + \int_{0}^{1} (x - x^3)\,\mathrm{d}x = 2\int_{0}^{1} (x - x^3)\,\mathrm{d}x = 2\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_{x=0}^{x=1} = \boxed{\frac{1}{2}}.$$

(b) (Final, 2014) The finite region bounded by the two curves $y = \sqrt{2}\cos(x\pi/4)$ and $y = |x|$.

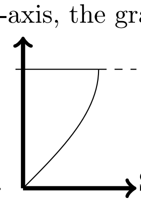


Solution: We draw a sketch first. We conclude that the area is

$$\begin{aligned} 2\int_{0}^{1} \left(\sqrt{2}\cos(x\pi/4) - x\right)\mathrm{d}x &= 2\left[\sqrt{2}\frac{4}{\pi}\sin\frac{\pi x}{4} - \frac{x^2}{2}\right]_{x=0}^{x=1} \\ &= \frac{8\sqrt{2}}{\pi}\cdot\frac{1}{\sqrt{2}} - 2 = \boxed{\frac{8}{\pi} - 2}. \end{aligned}$$

(2) Find the total area of the following planar regions. It will be useful to sketch the region first.

(a) The finite region bounded by the y-axis, the graph of $y = \arcsin(x)$ and the line $y = \frac{\pi}{2}$.



Solution: We draw a sketch first. Slicing vertically, requires evaluating

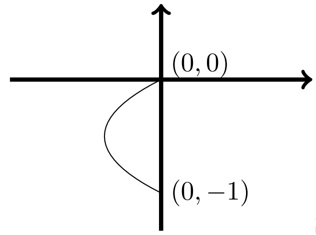
$$\int_{x=0}^{x=1} (1 - \arcsin x)\,\mathrm{d}x$$

which is painful. Slicing horizontally instead, we have $0 \le y \le \frac{\pi}{2}$ and at each y the length of the slice is $x = \sin y$ so instead we compute

$$\int_{y=0}^{y=\pi/2} \sin y\,\mathrm{d}y = [-\cos y]_{y=0}^{y=\pi/2} = 1\,.$$

Date: 18/1/2017, Worksheet by Lior Silberman. This instructional material is excluded from the terms of UBC Policy 81.

(b) (Quiz, 2015) The finite region to the left of the y-axis and to the right of the curve $x = y^2 + y$.



Solution: We draw a sketch first. Slicing horizontally, we have $-1 \leq y \leq 0$ and for each y, the slice has length $-(y^2 + y)$. The area is therefore

$$\int_{y=-1}^{y=0} (-y^2 - y)\,\mathrm{d}y = -\left[\frac{y^3}{3} + \frac{y^2}{2}\right]_{y=-1}^{y=0} = \frac{(-1)^3}{3} + \frac{(-1)^2}{2} = \boxed{\frac{1}{6}}.$$